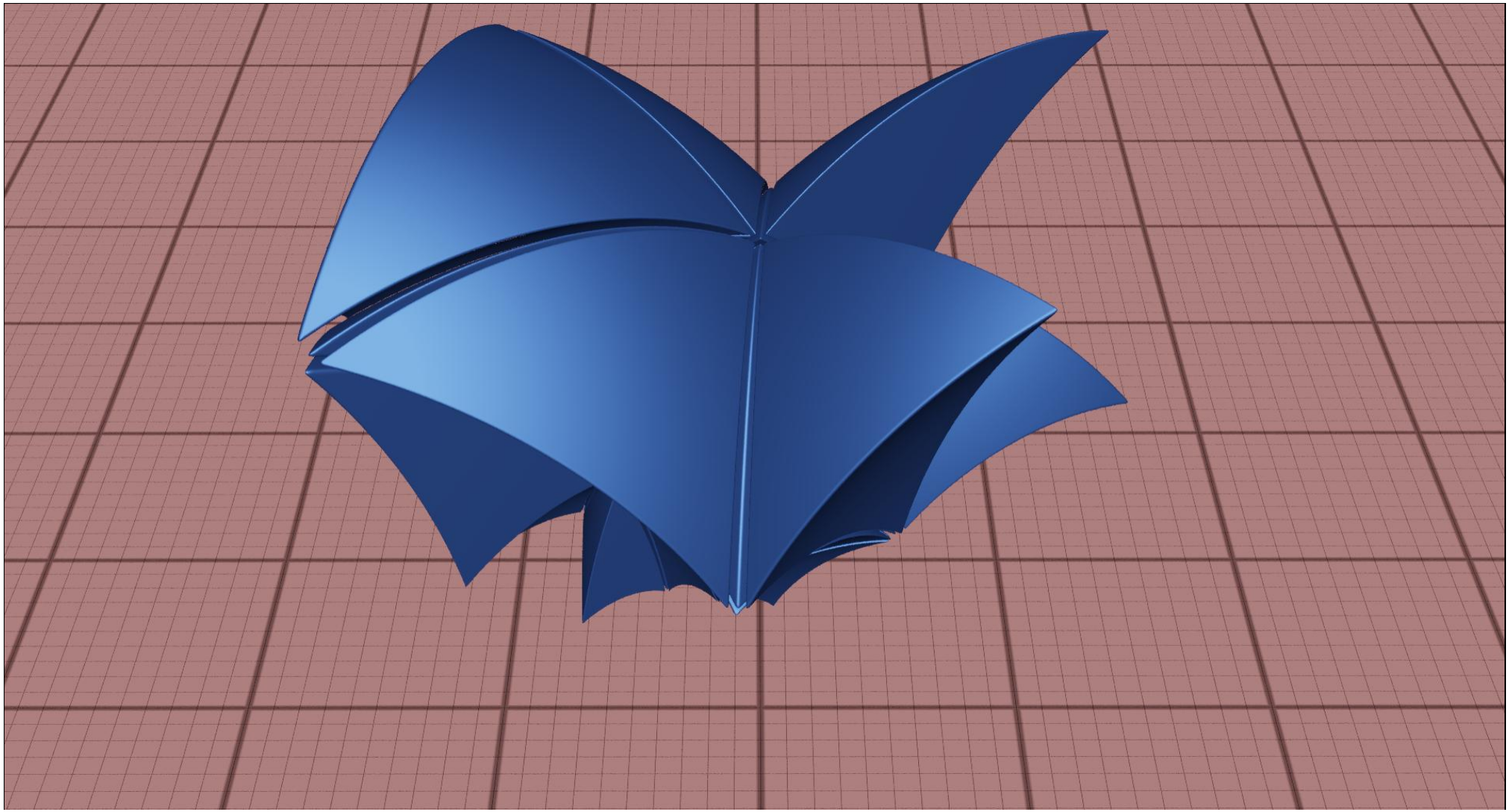
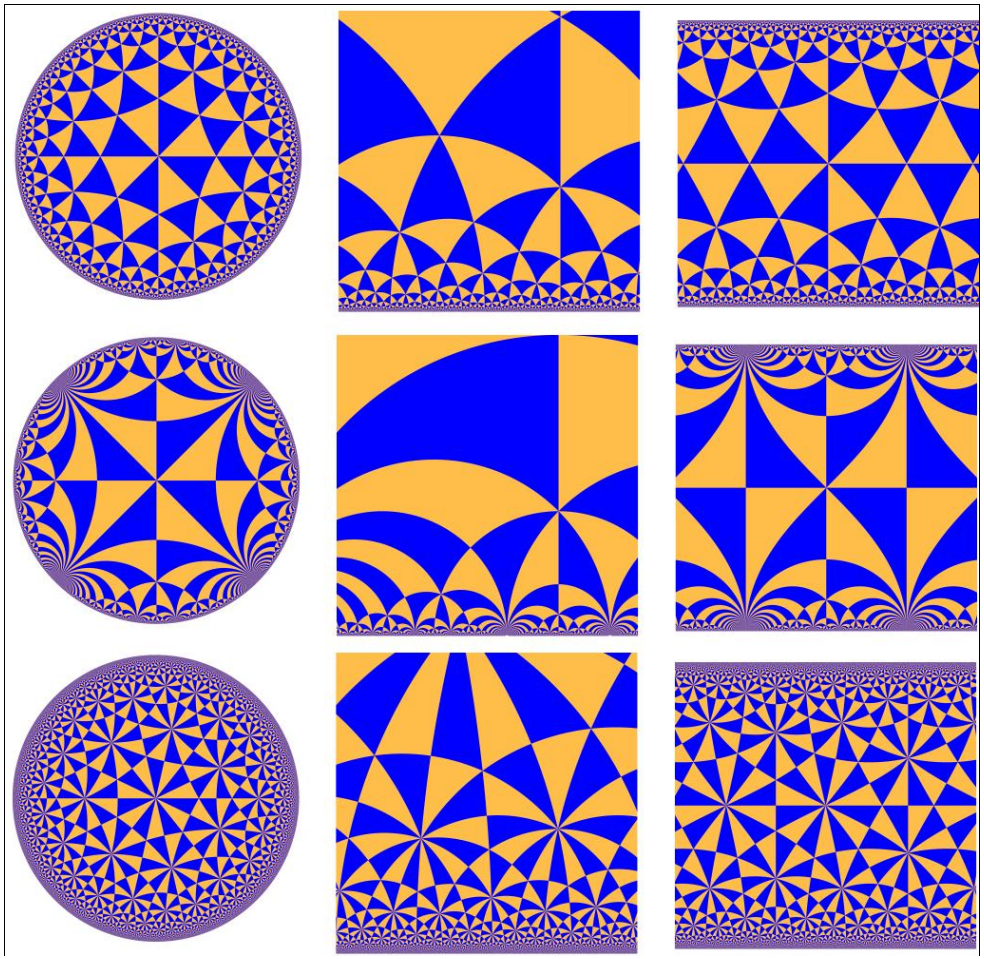
Cross sections of 3D hyperbolic tilings

Vladimir Bulatov, Shapeways

G4G13, Atlanta, April 14, 2018



2D hyperbolic tilings



Tiling by triangles are represented in

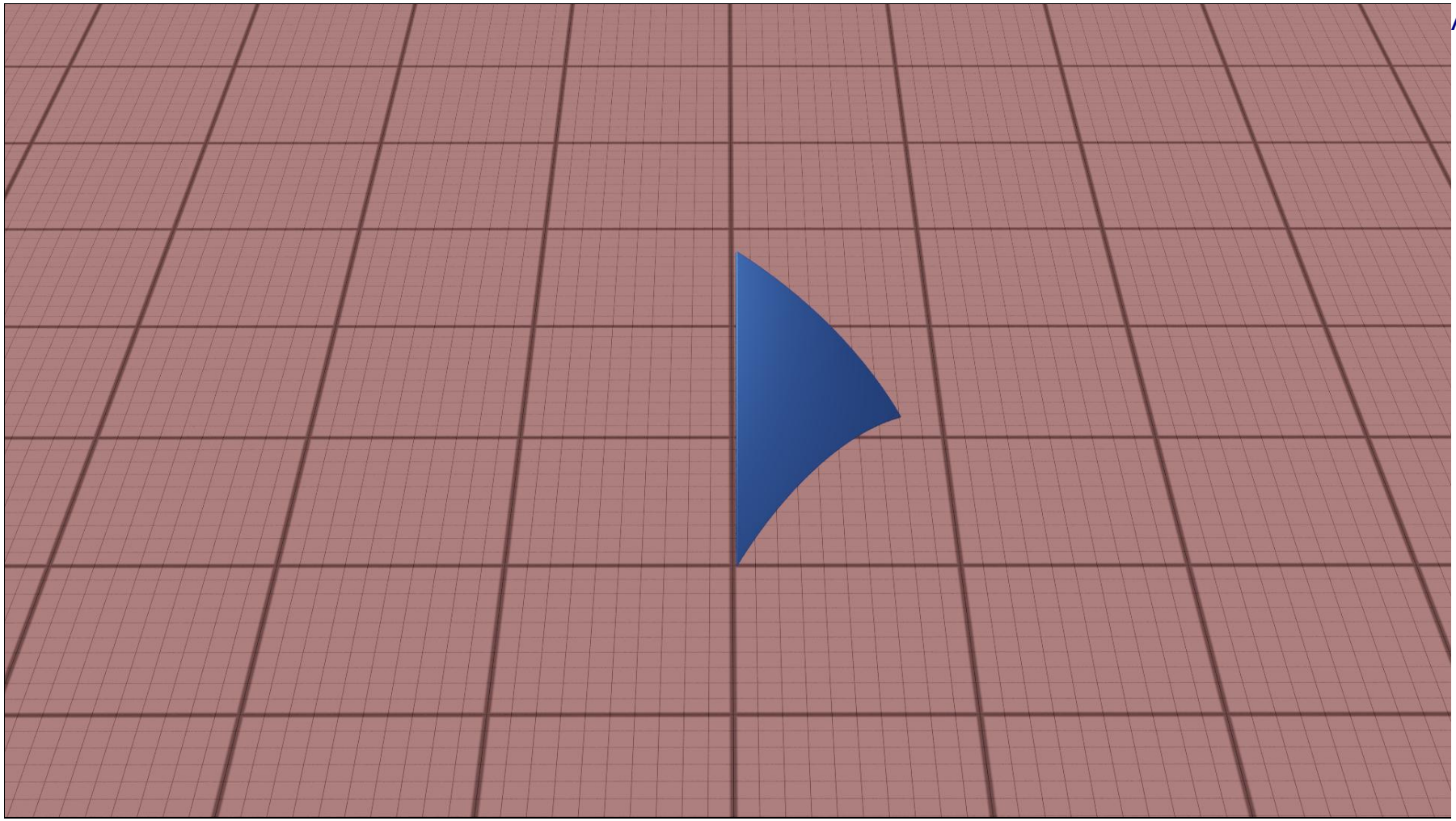
- Poincare Circle Model (left)
- Upper Half Plane Model (center)
- Band Model (right)

Simple to draw.

Easy to understand.

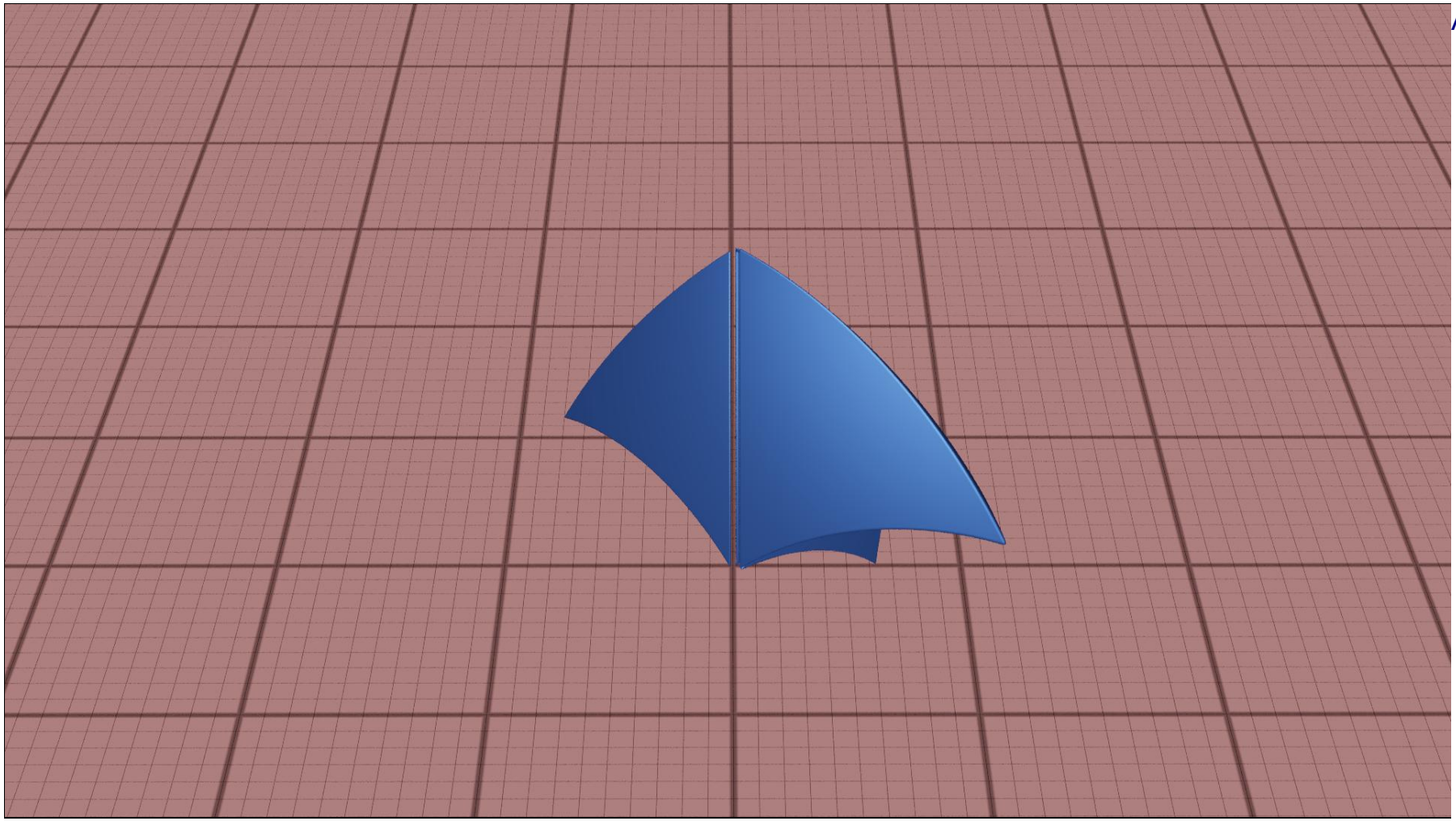
Similar properties in the different models - tiles size tends to zero as tiles approach the boundary of the model

3D hyperbolic tilings are hard to draw and understand



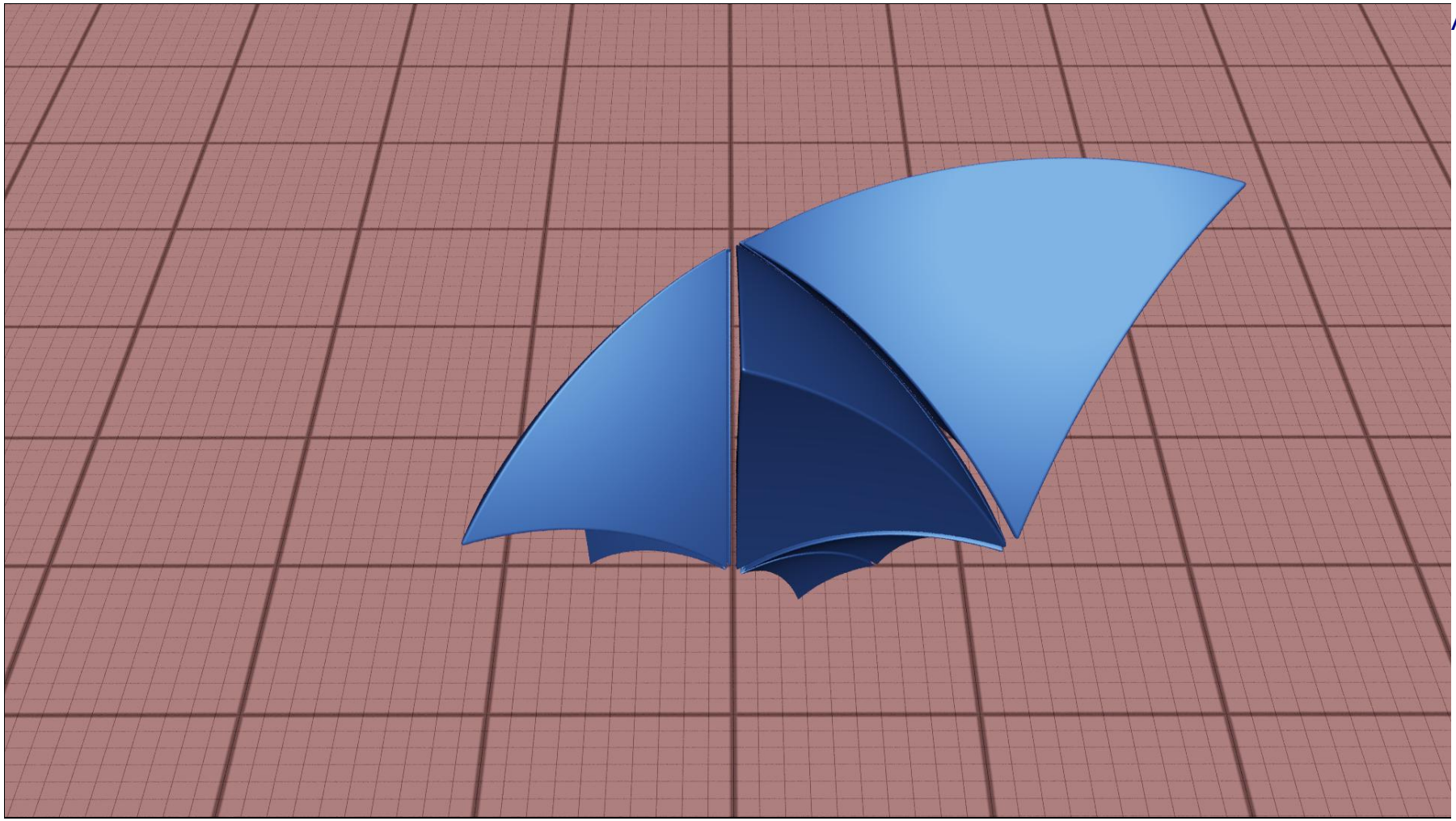
Animation

3D hyperbolic tilings are hard to draw and understand



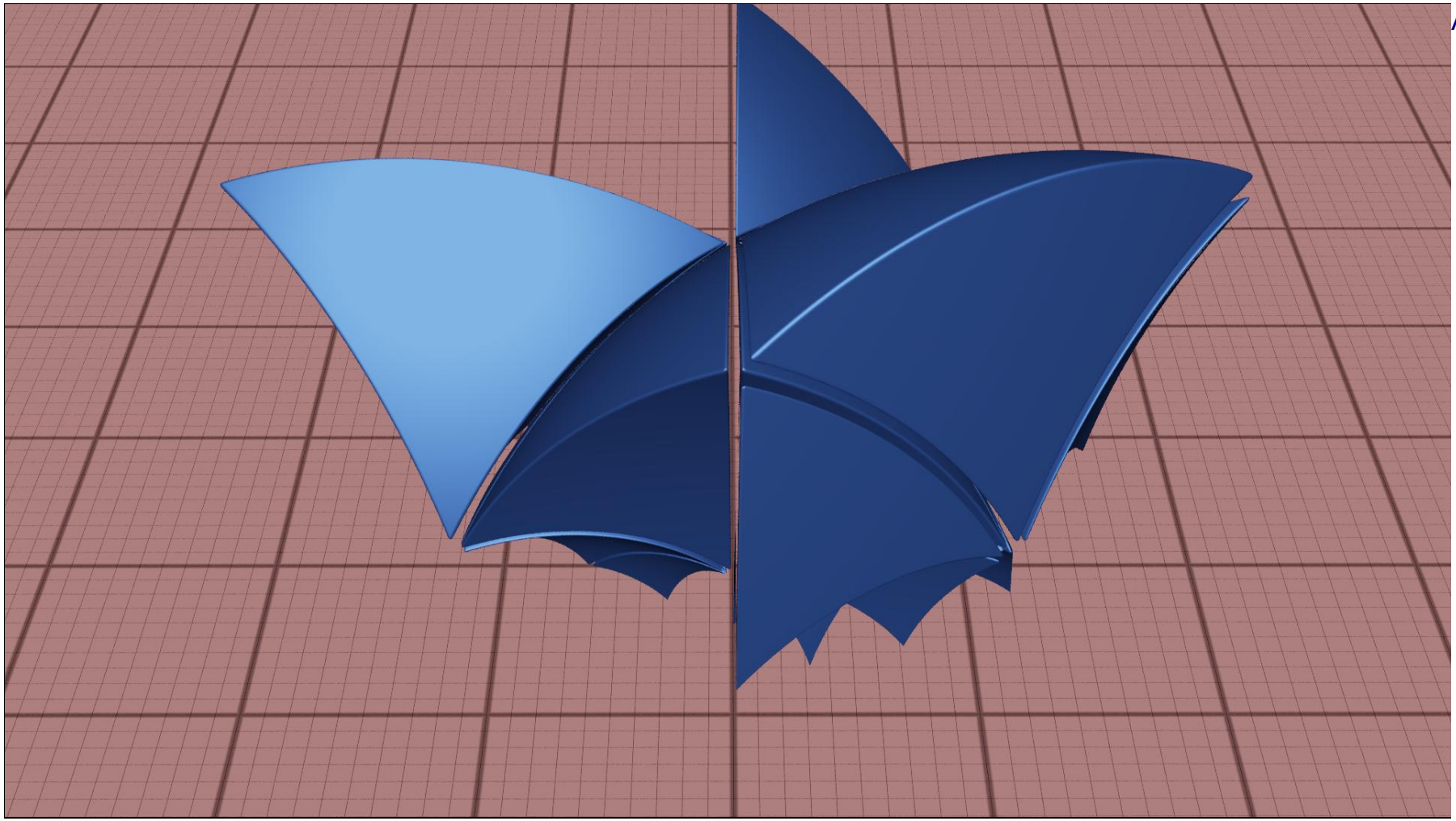
Animation

3D hyperbolic tilings are hard to draw and understand



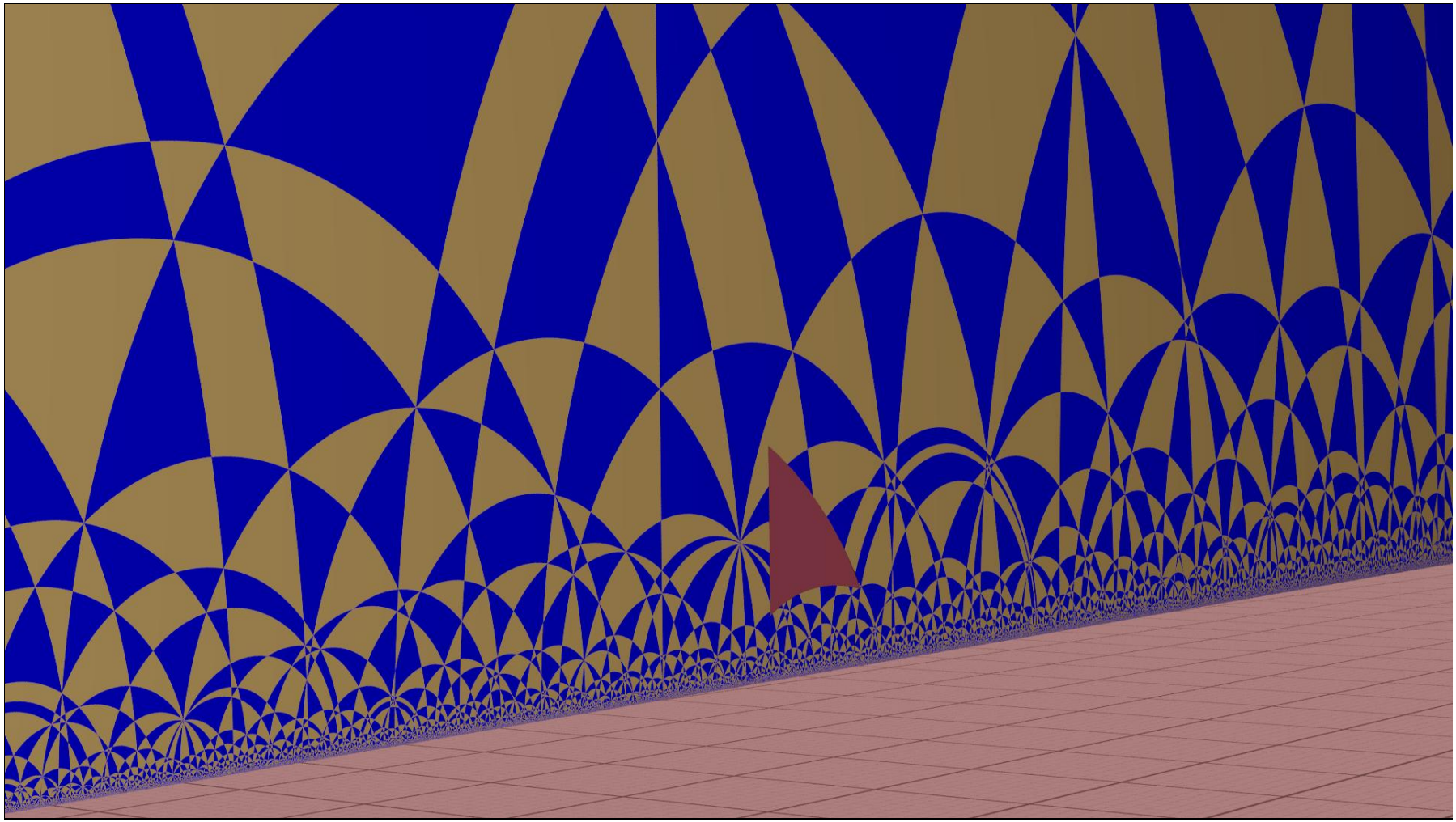
Animation

3D hyperbolic tilings are hard to draw and understand

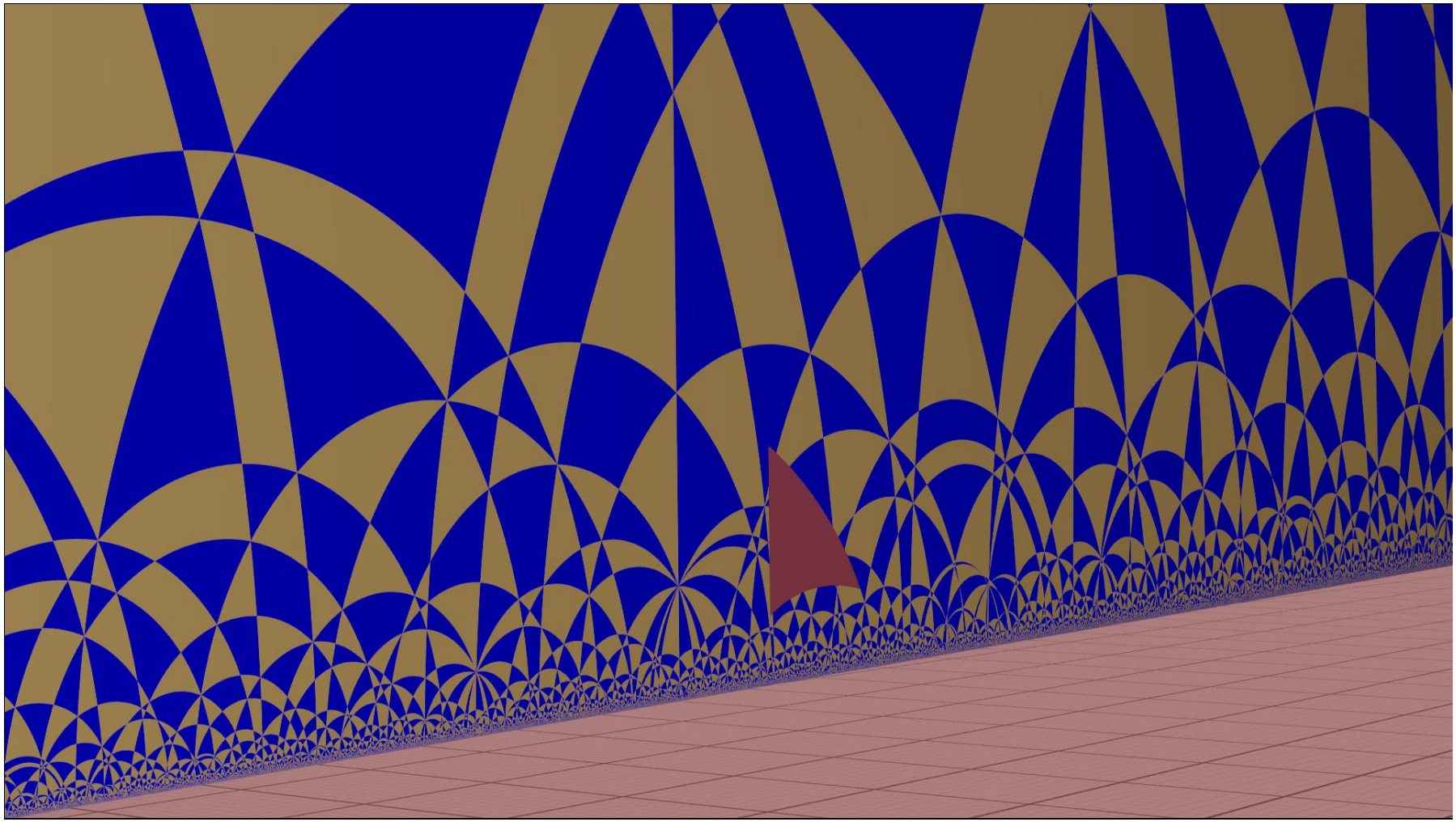


Animation

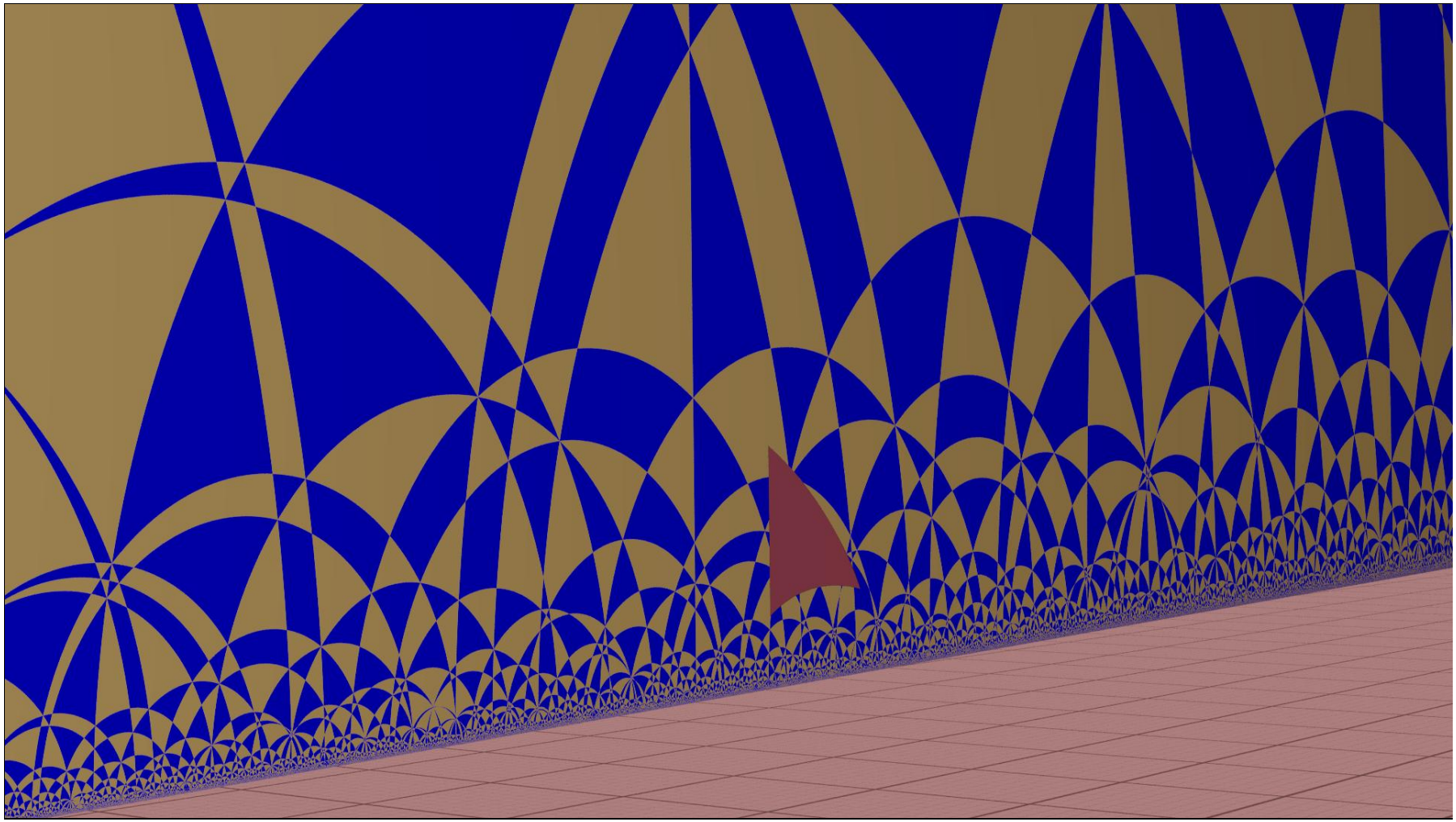
Plane cross section



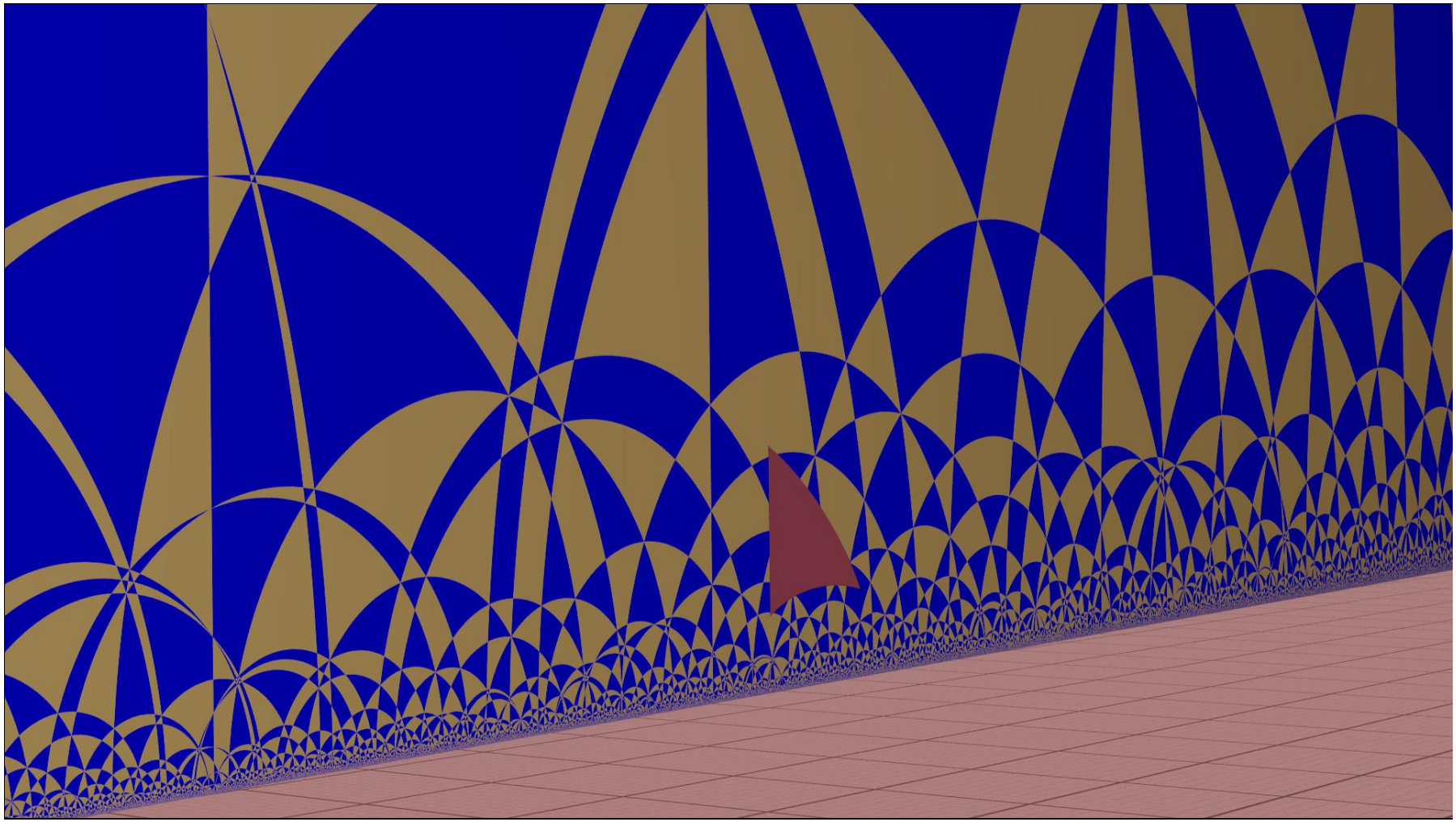
Plane cross section 1



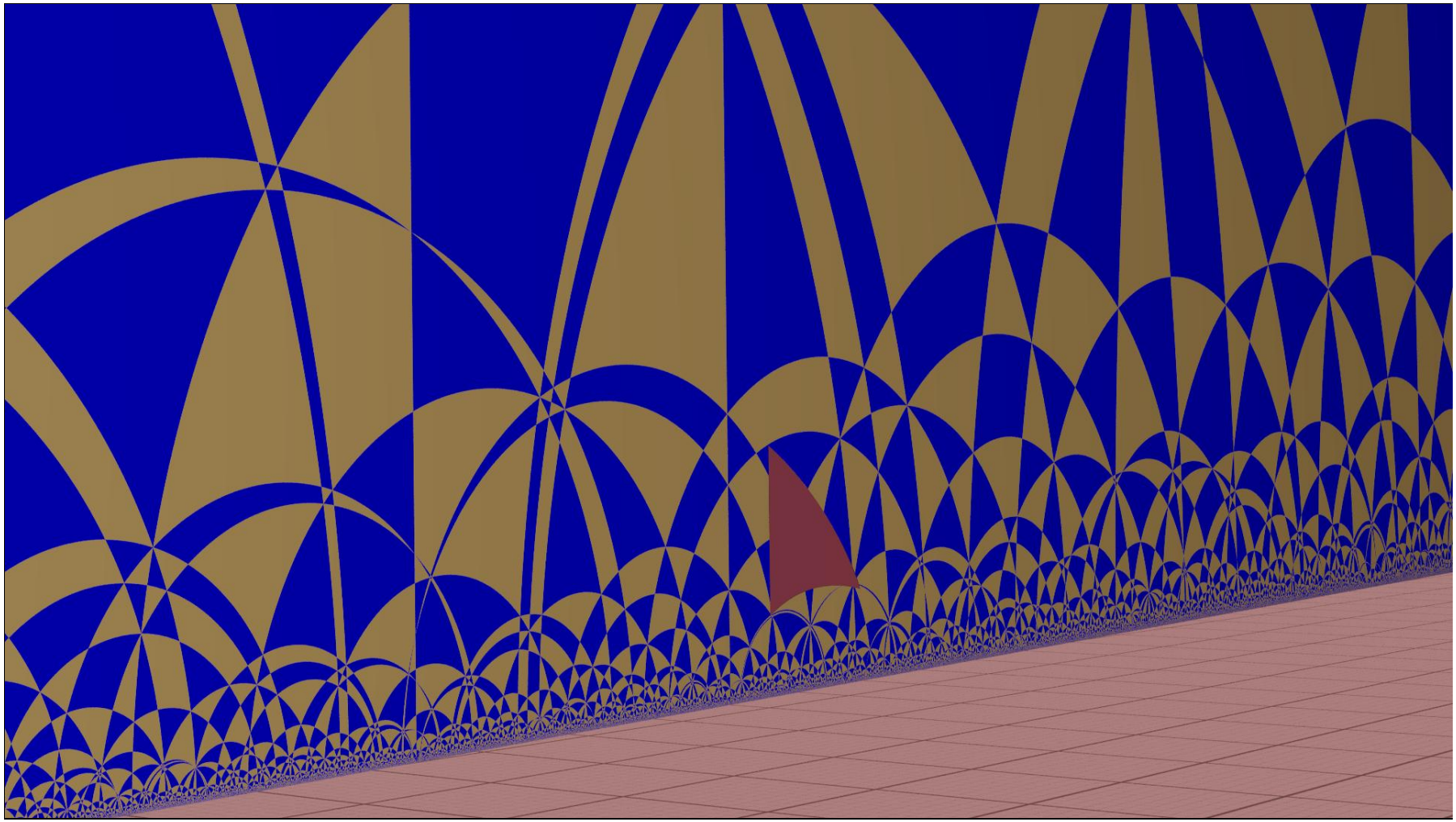
Plane cross section 2



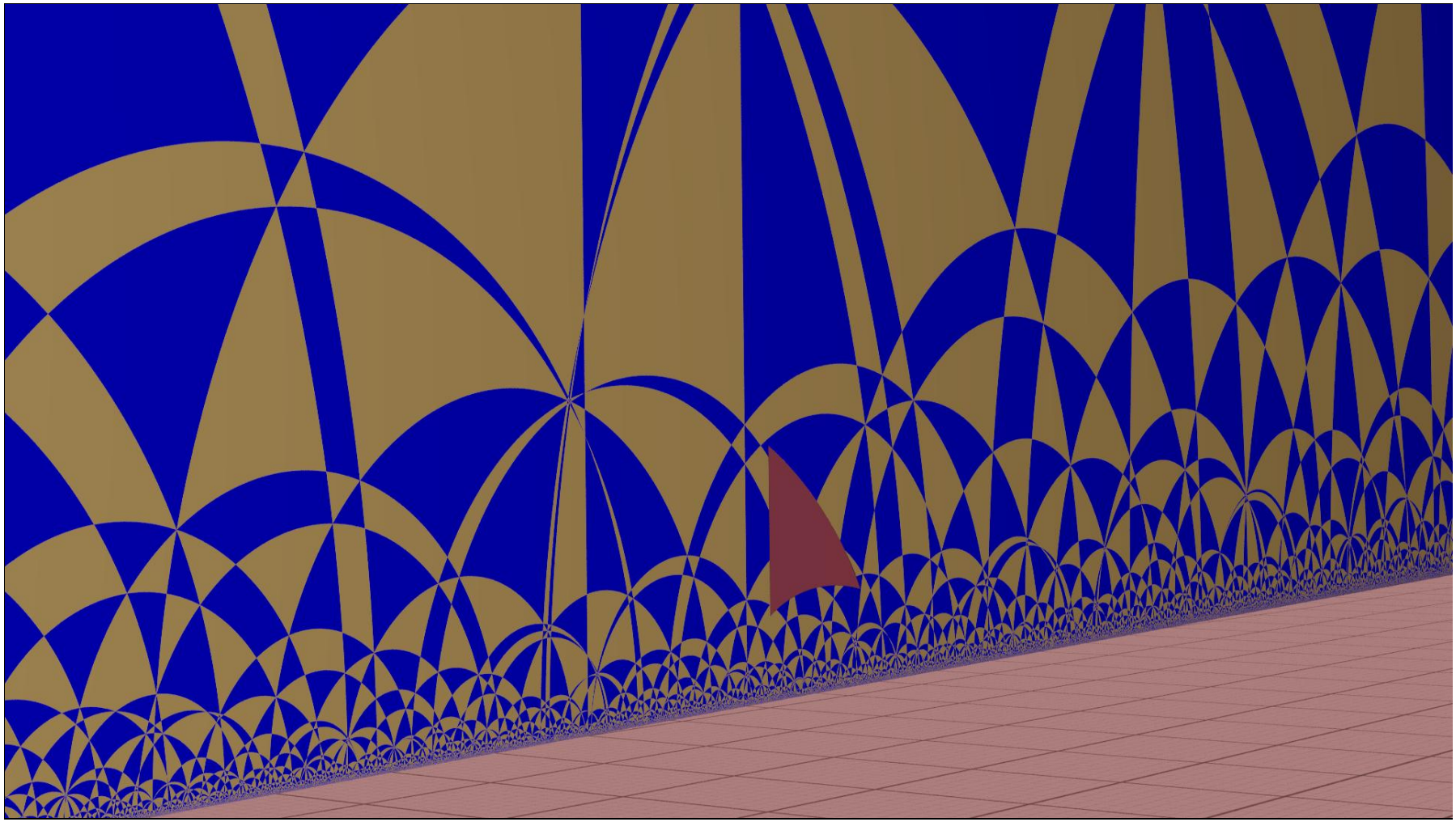
Plane cross section 3



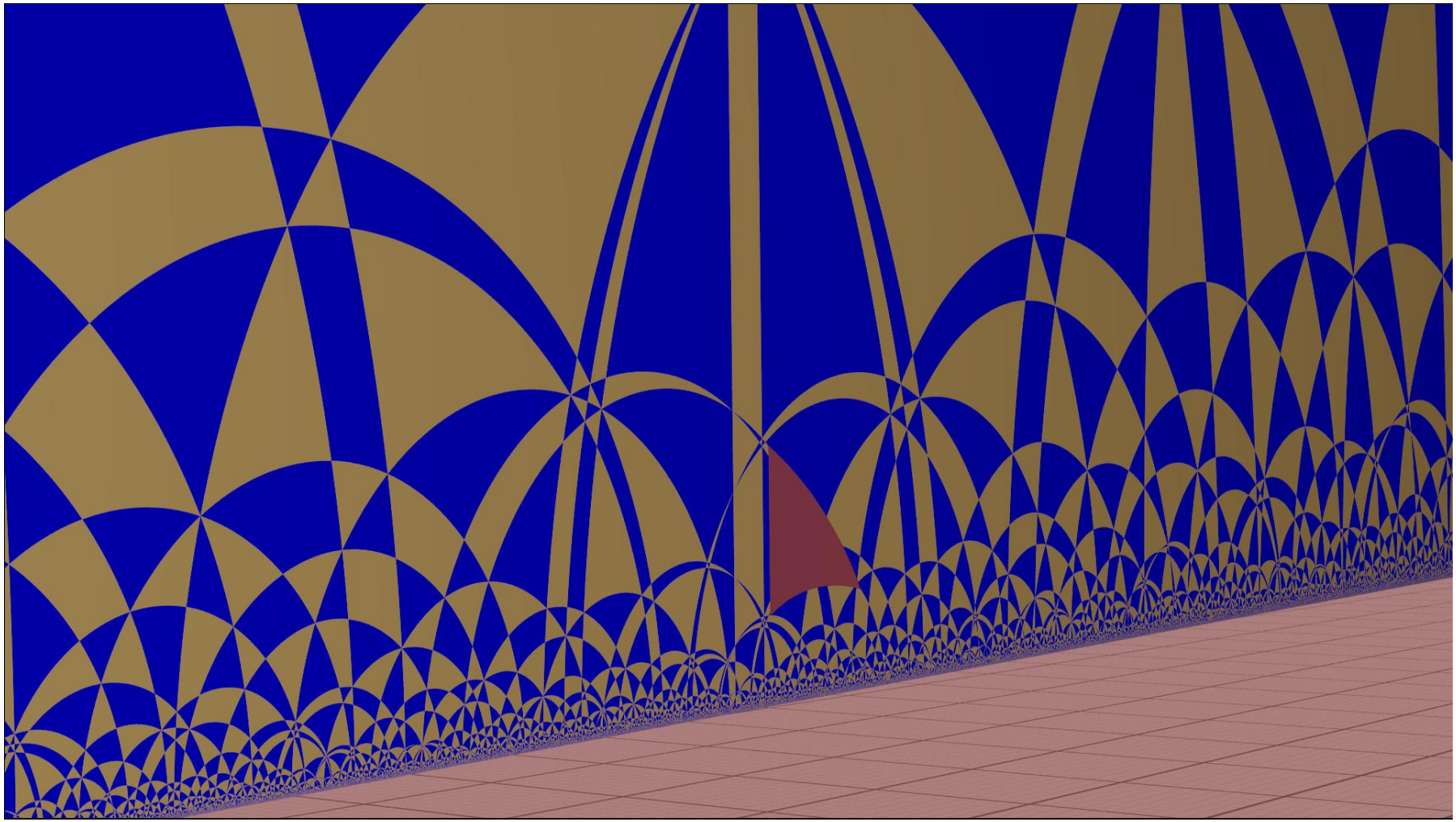
Plane cross section 4



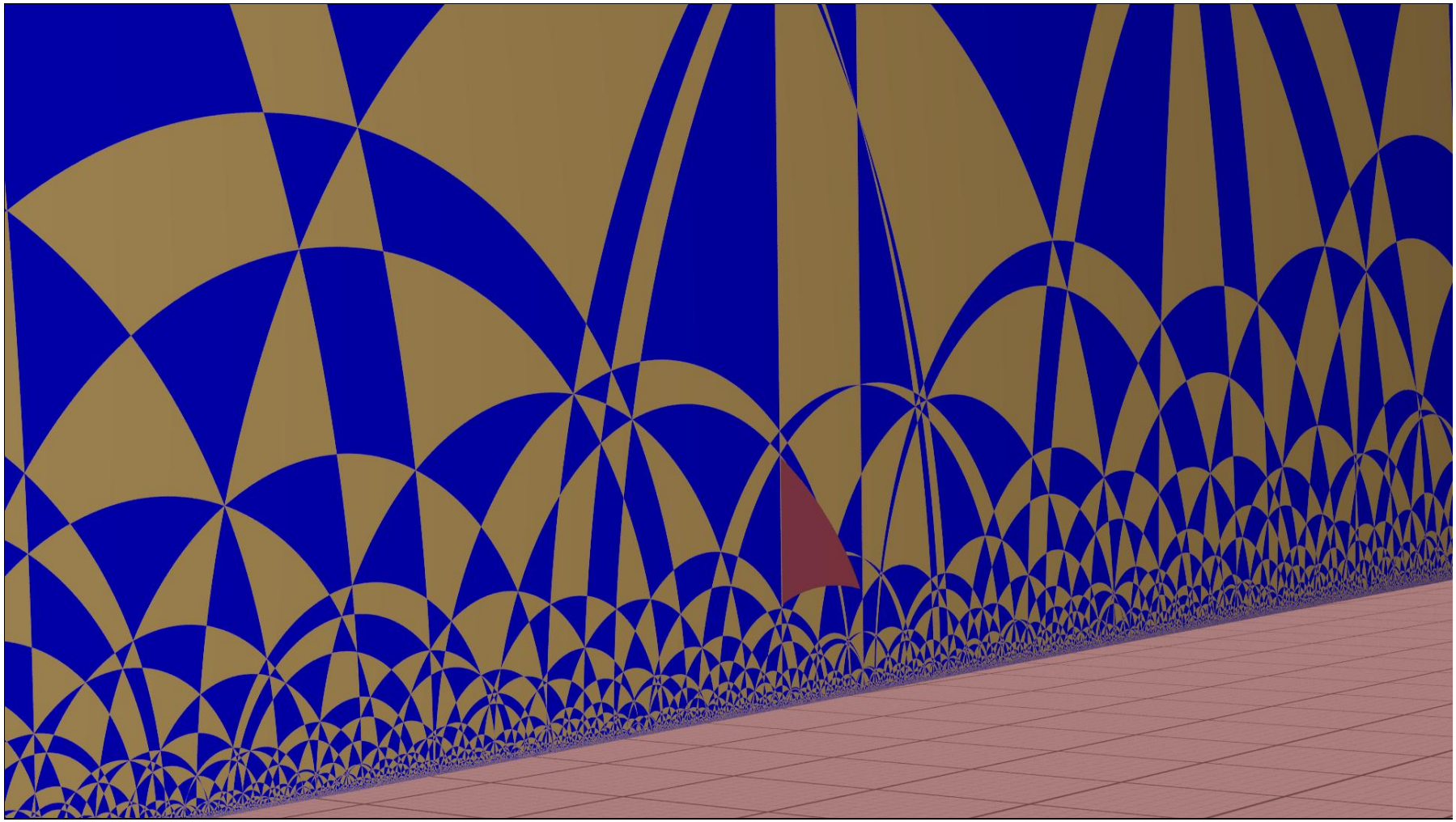
Plane cross section 5



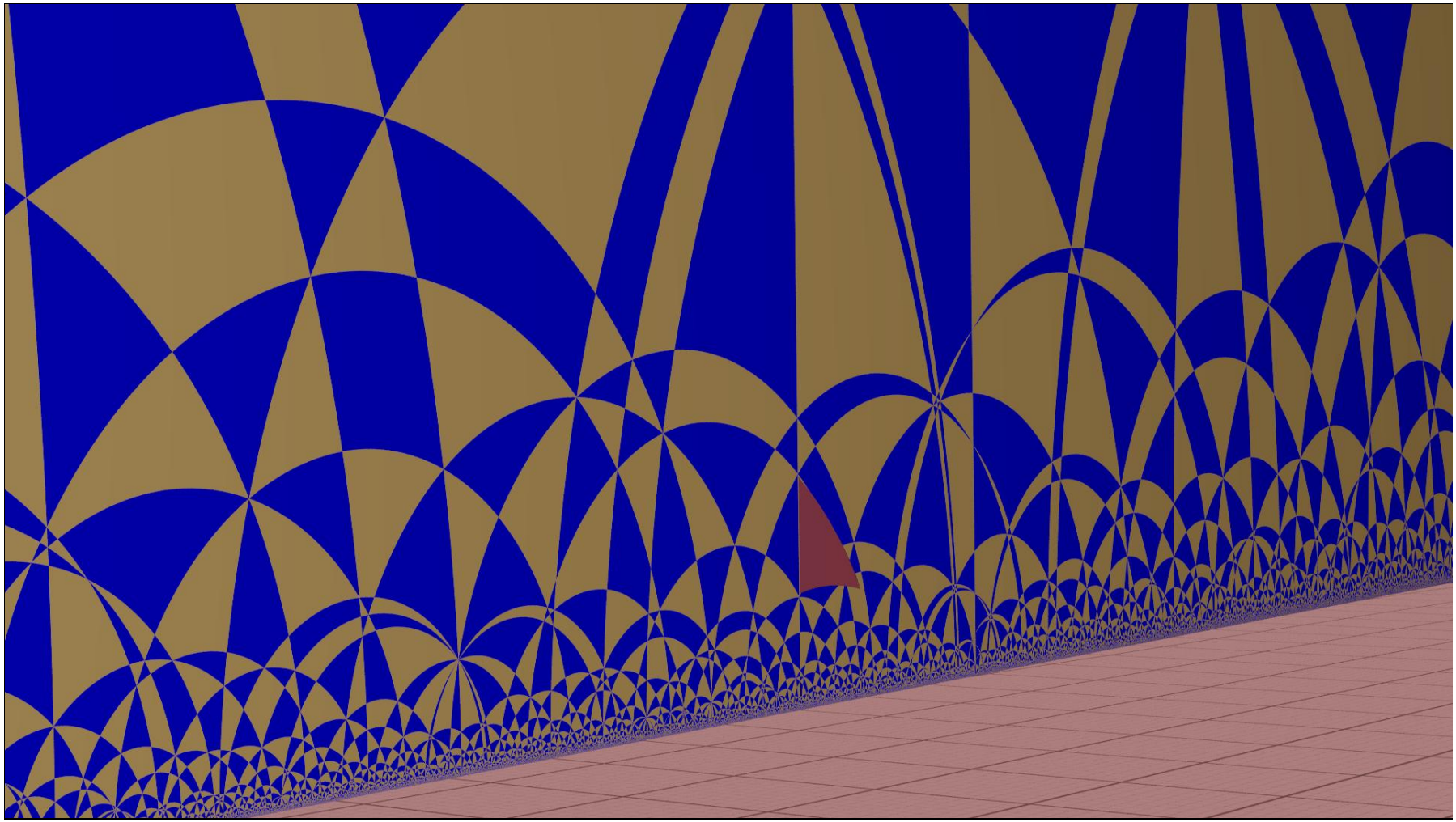
Plane cross section 6



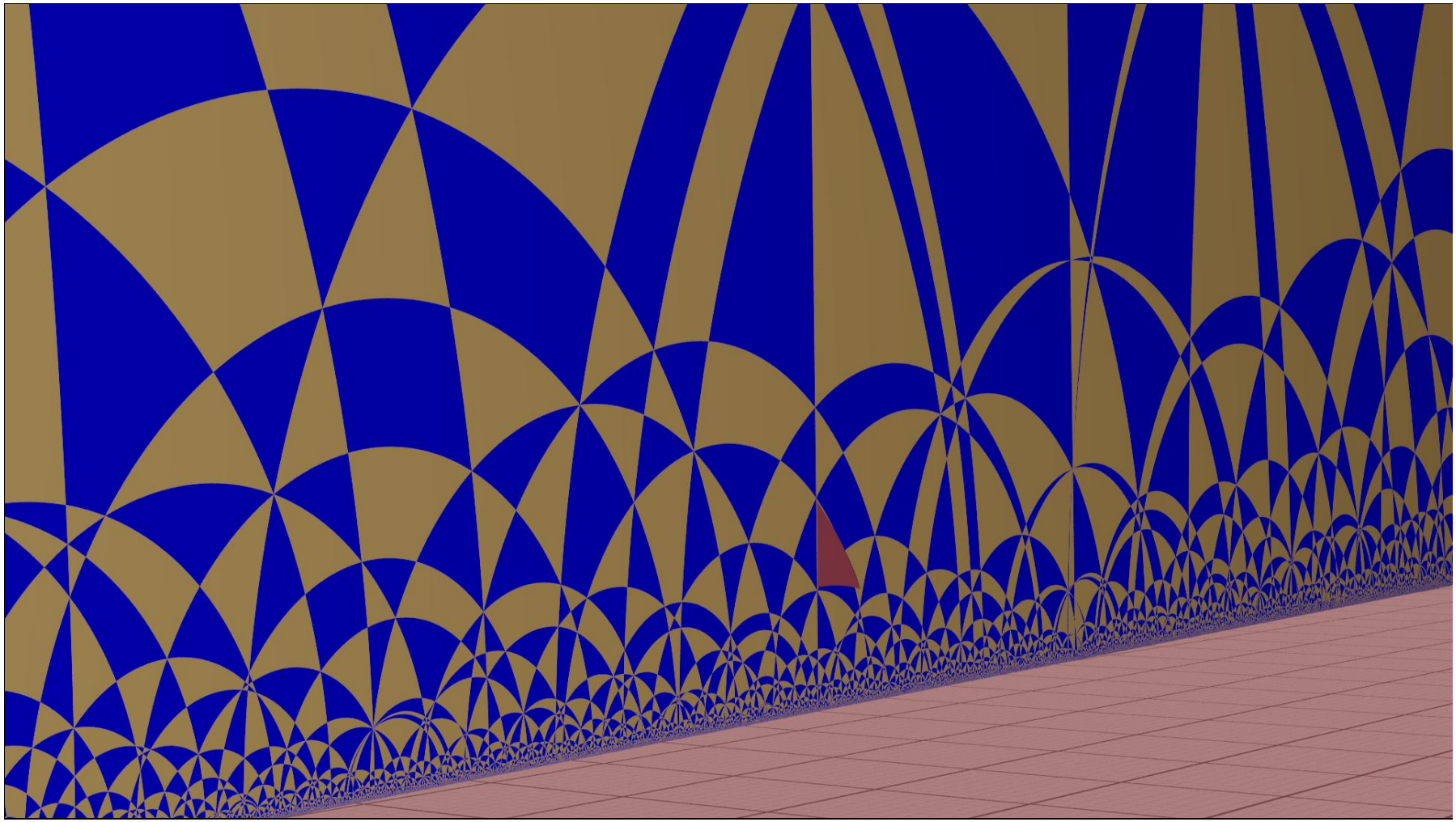
Plane cross section 7



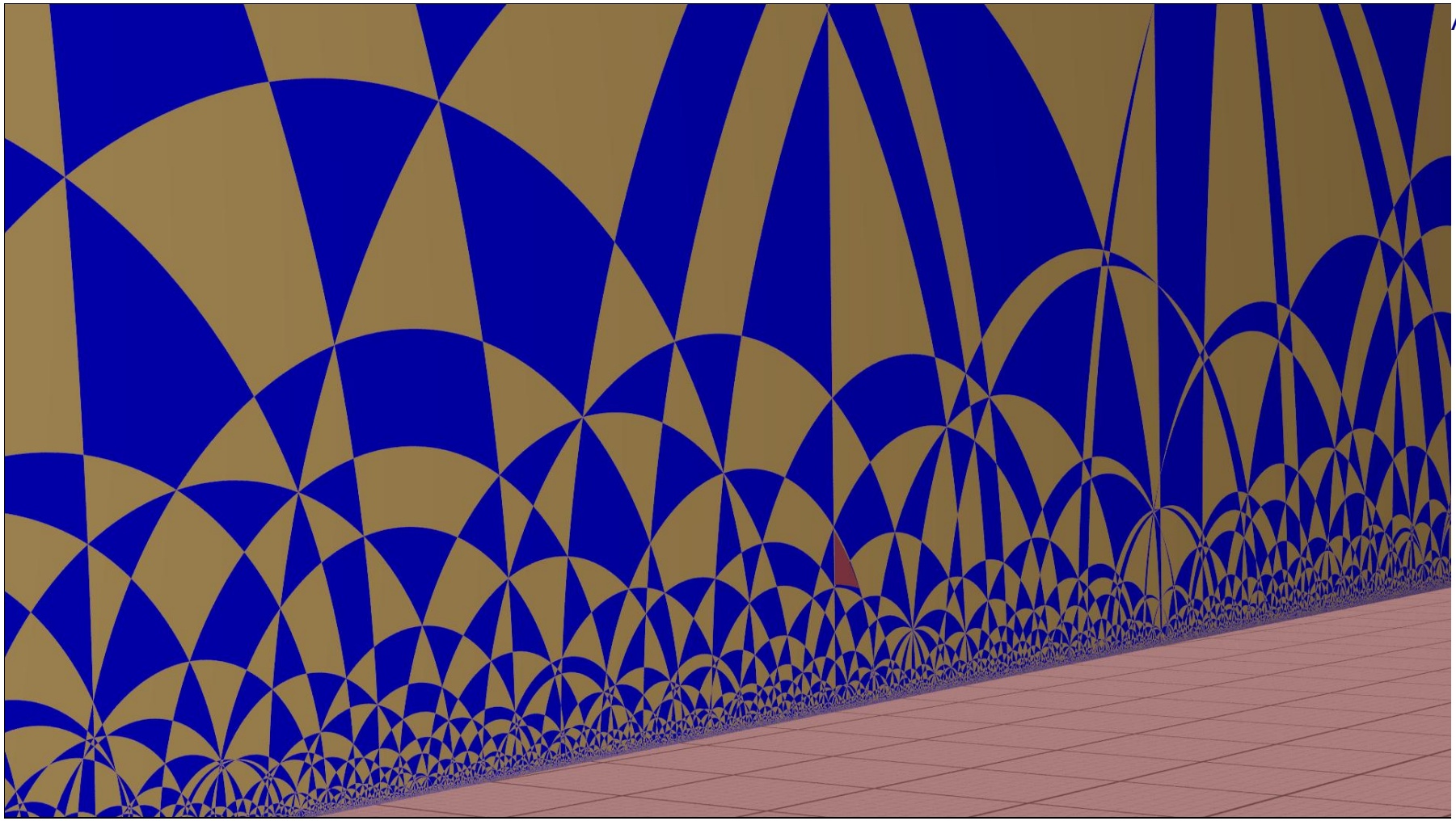
Plane cross section 8



Plane cross section 9

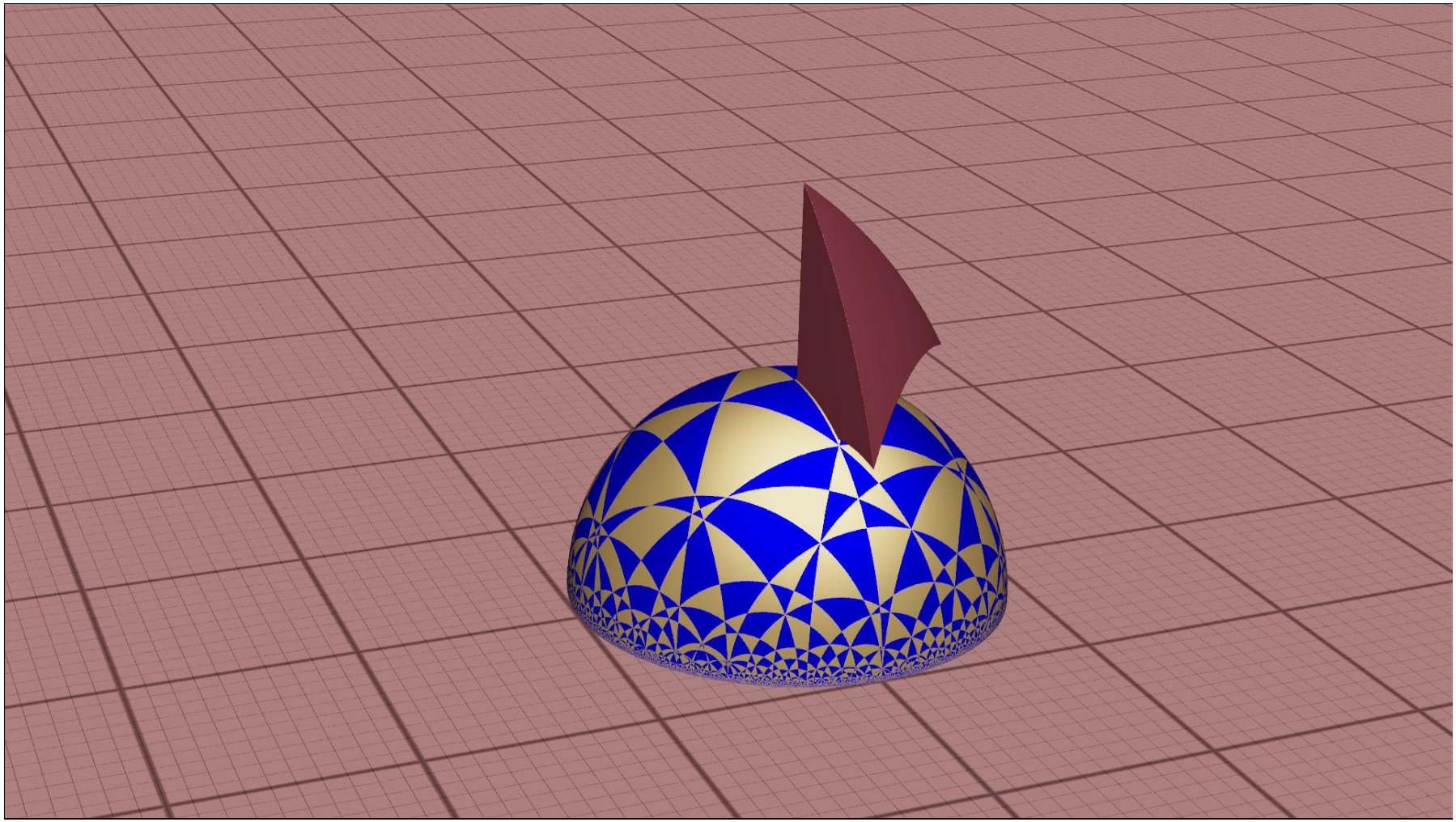


Plane cross section 10

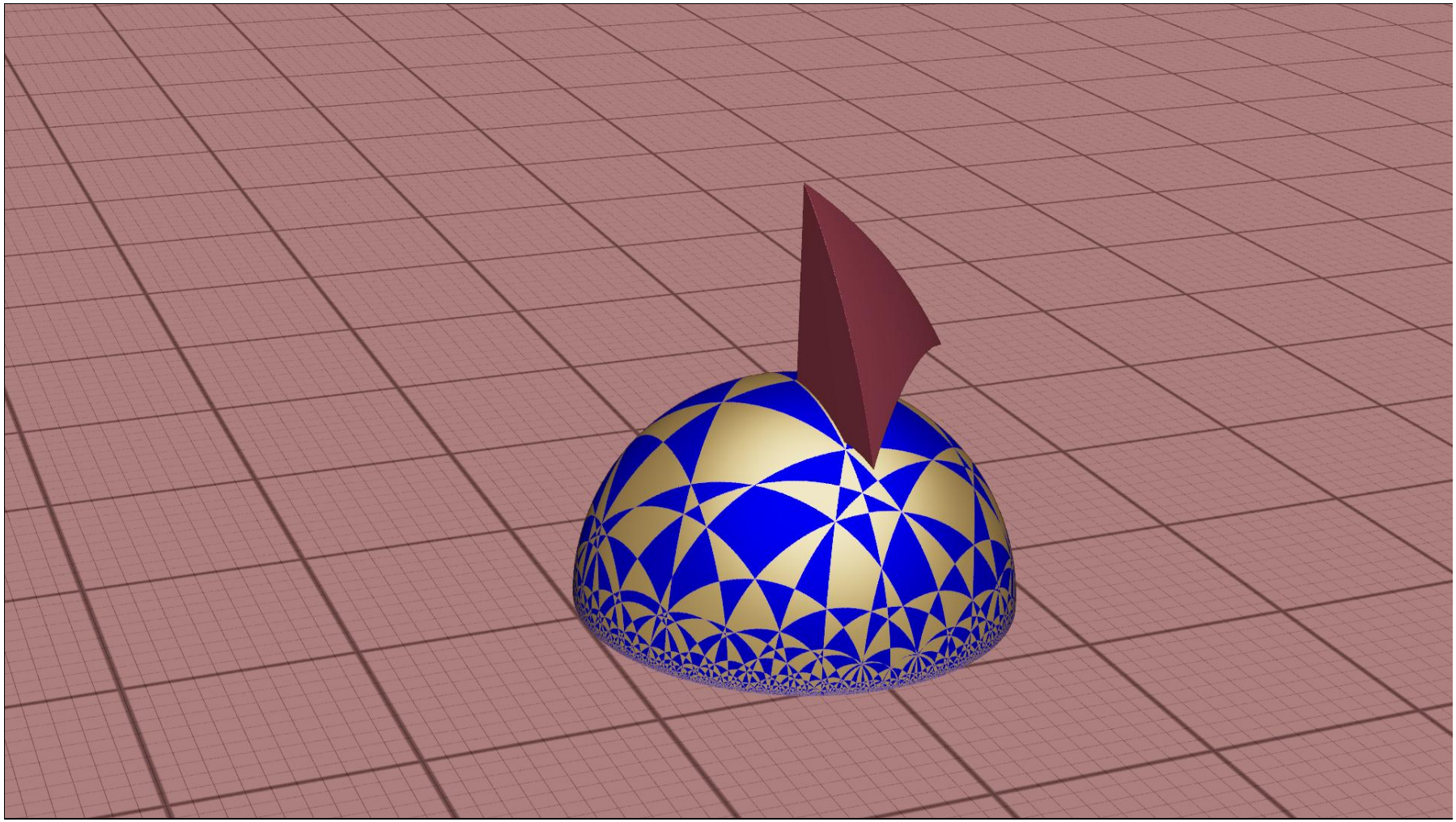


Animation

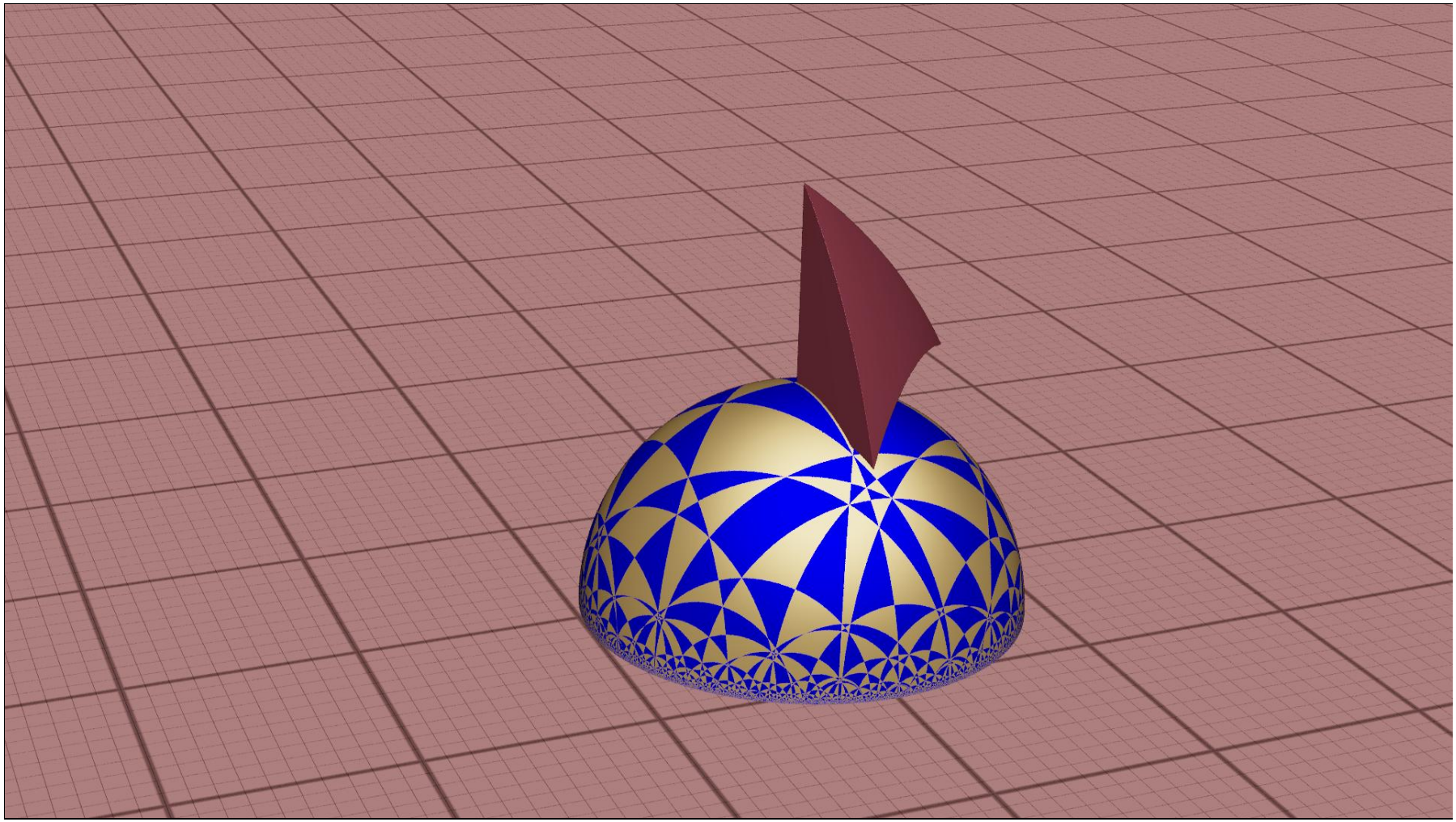
Half sphere cross section



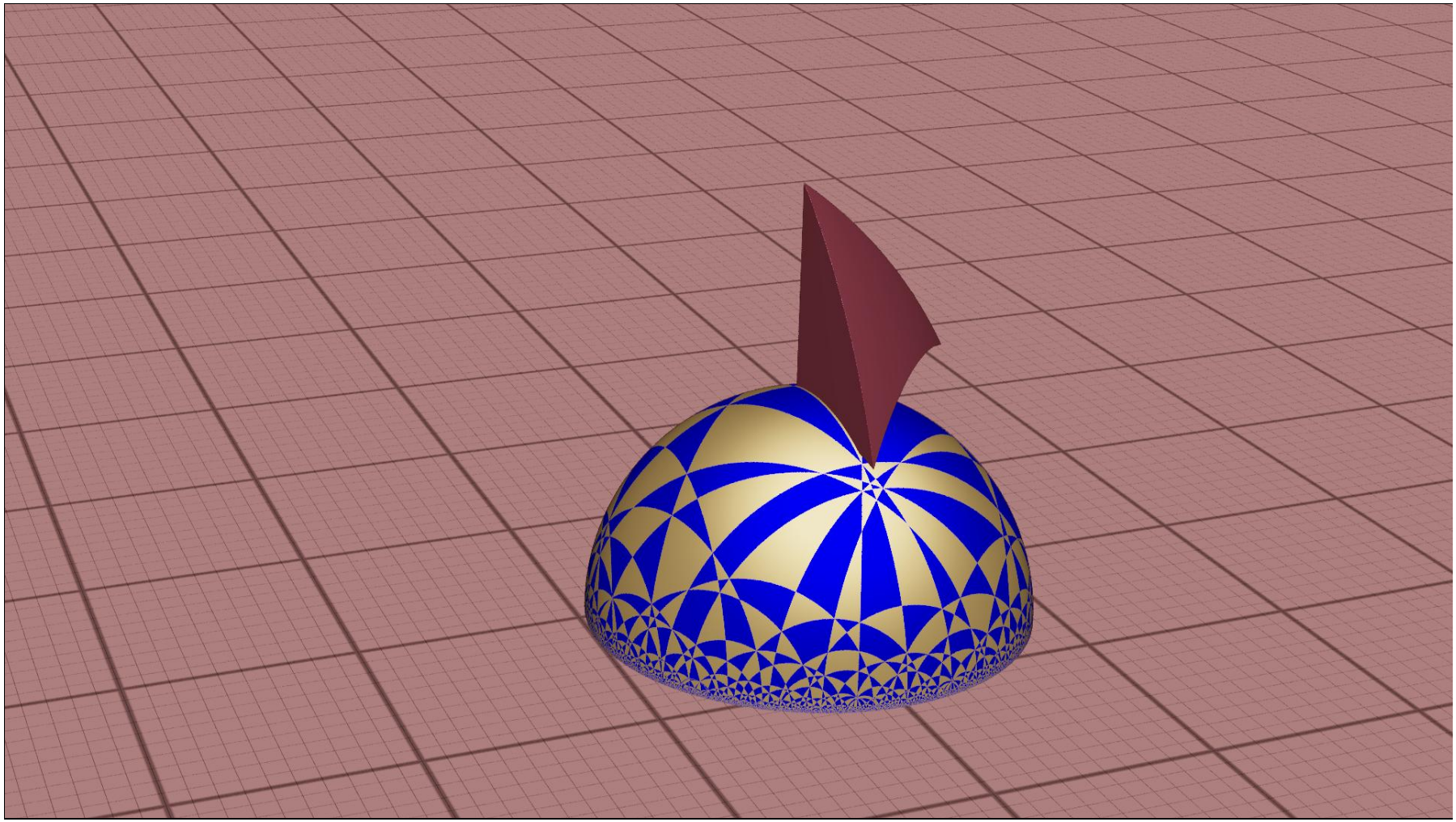
Half sphere cross section 1



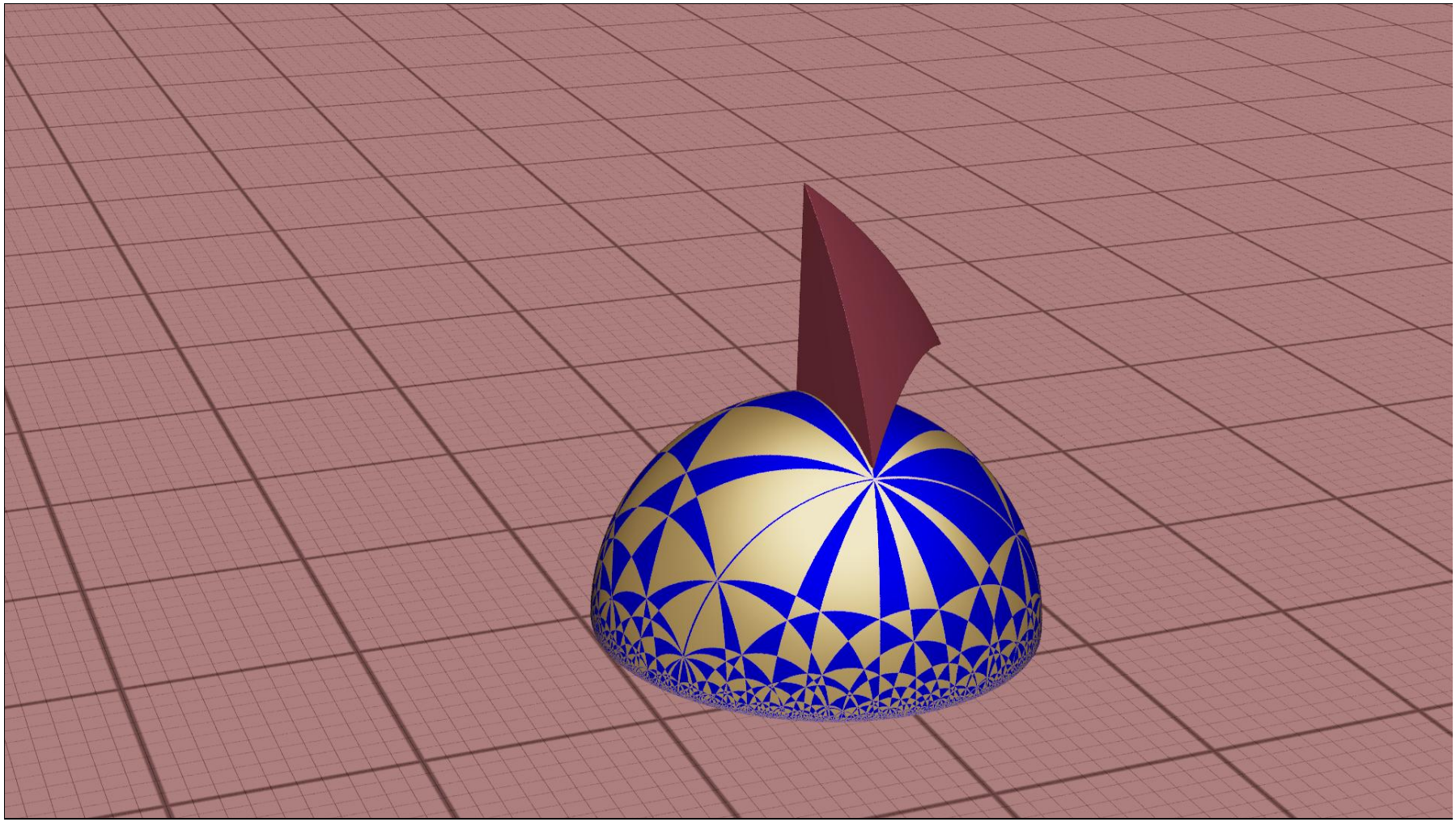
Half sphere cross section 2



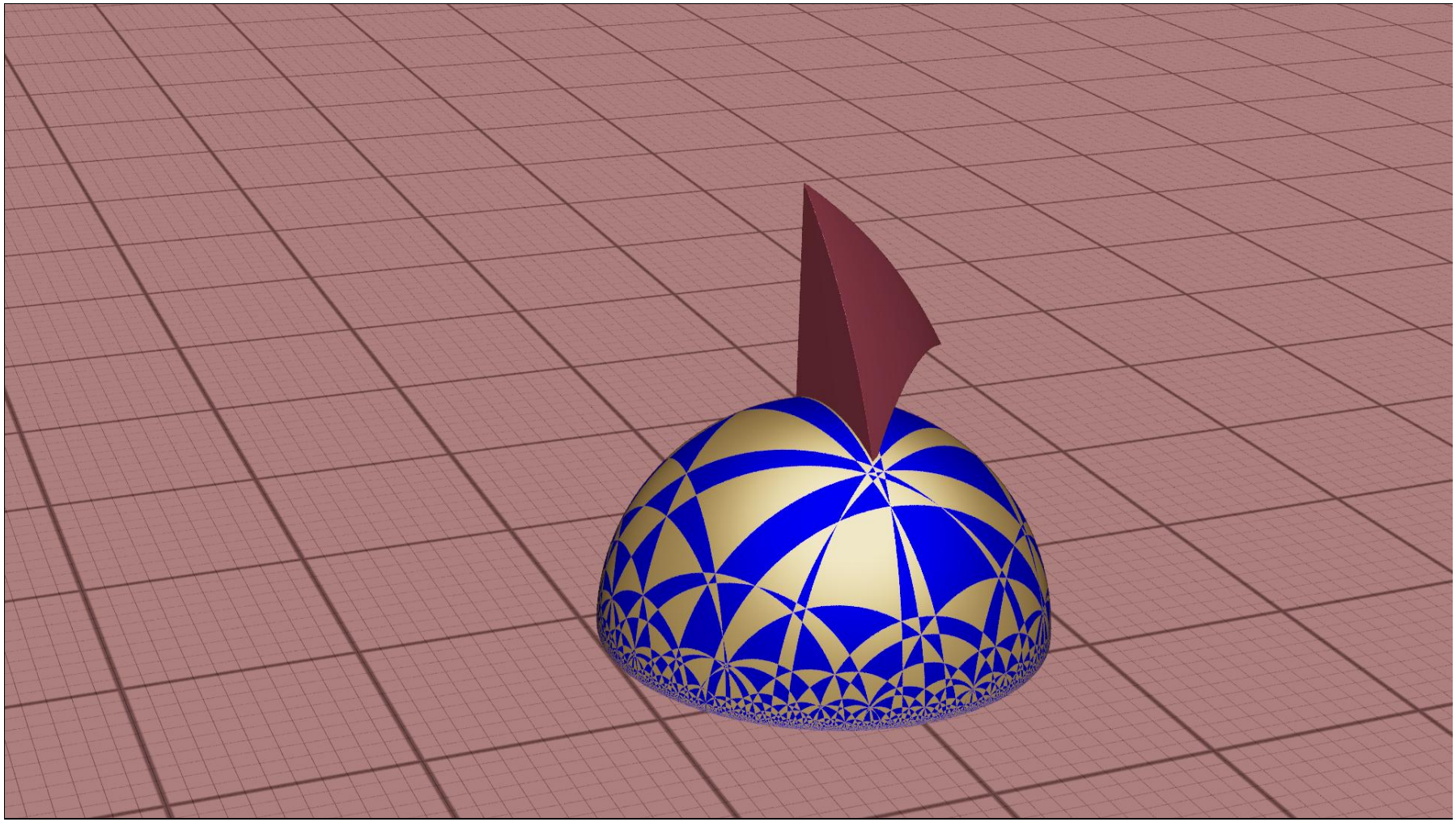
Half sphere cross section 3



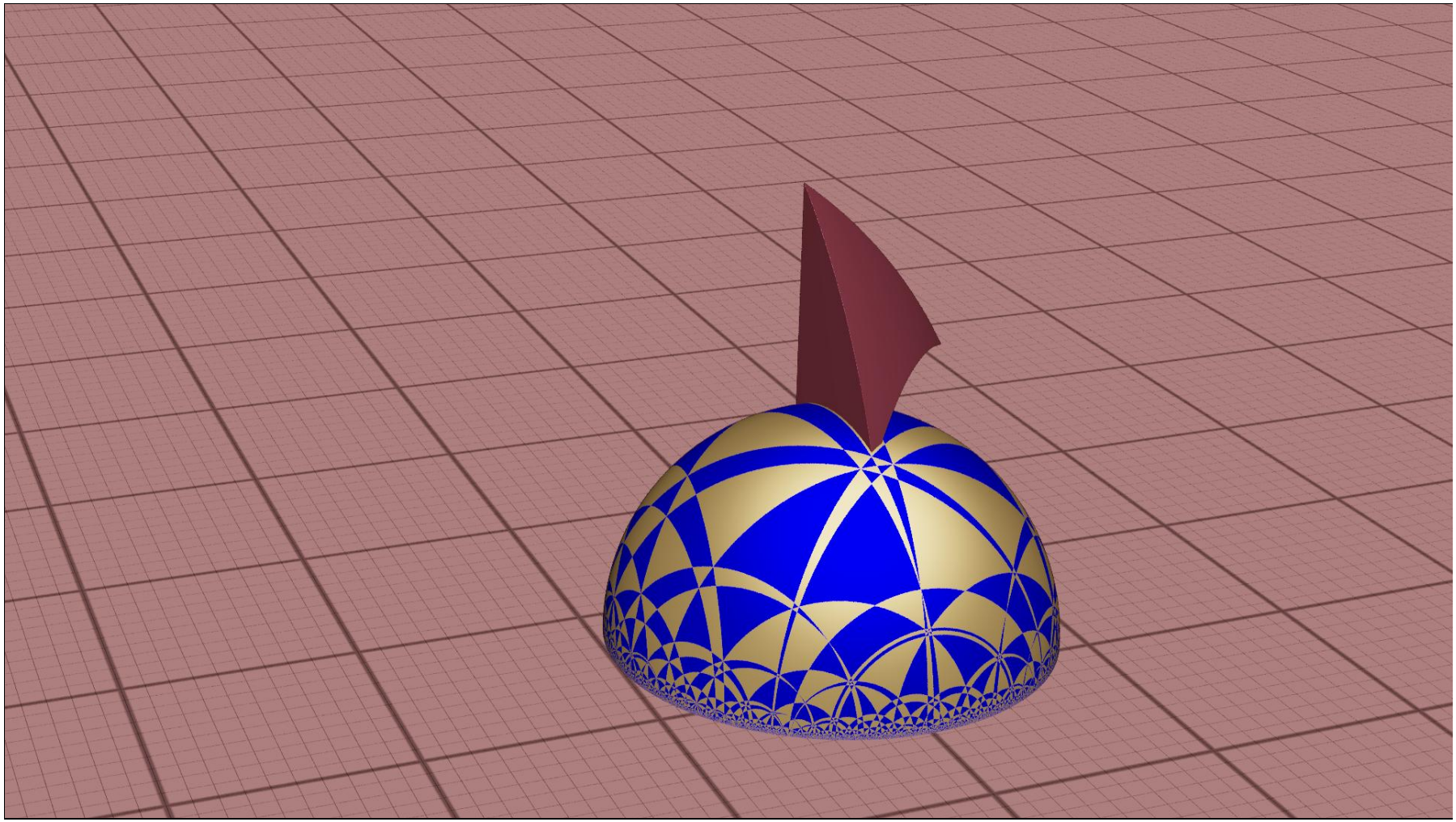
Half sphere cross section 4



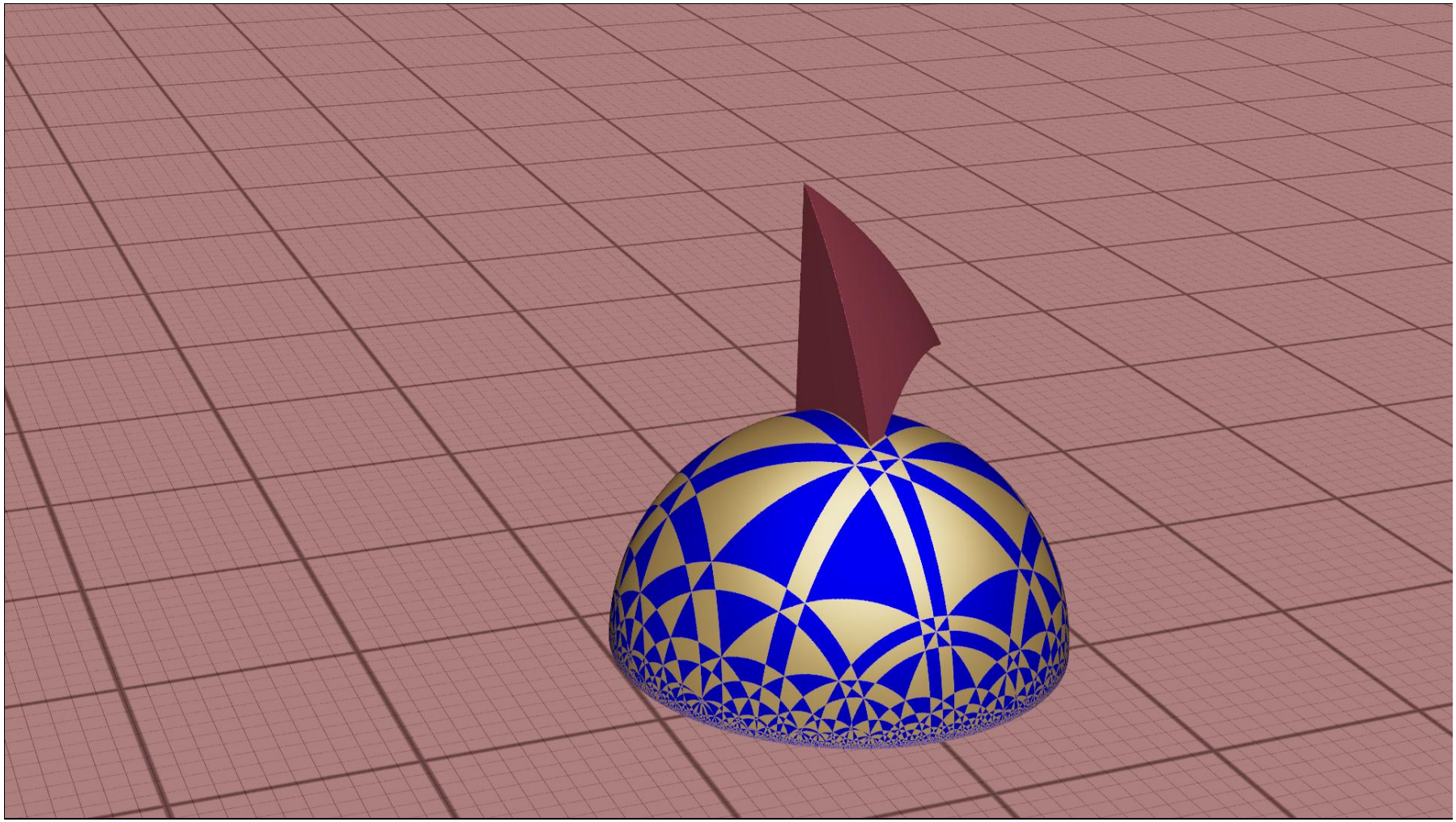
Half sphere cross section 5



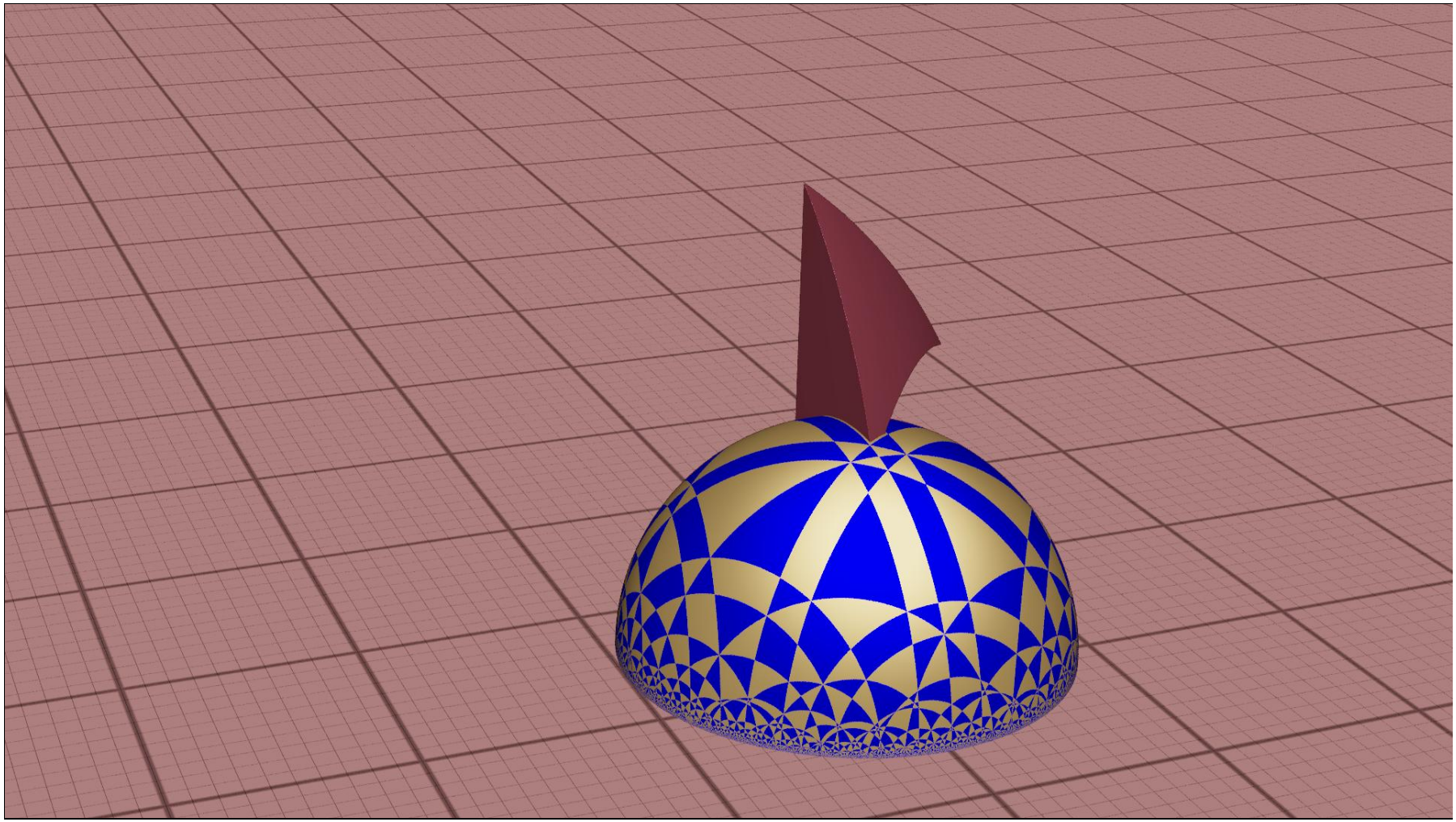
Half sphere cross section 6



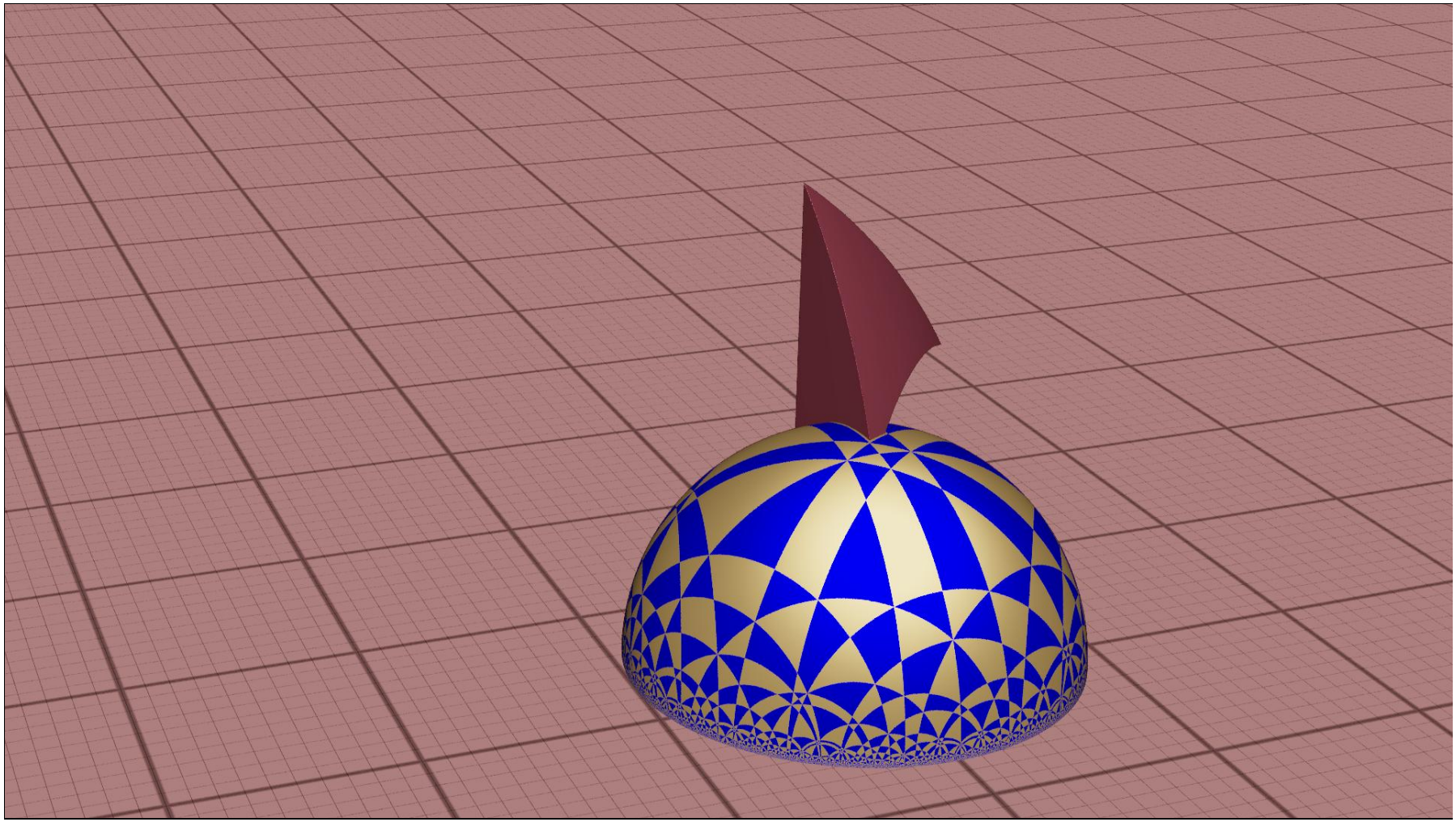
Half sphere cross section 7



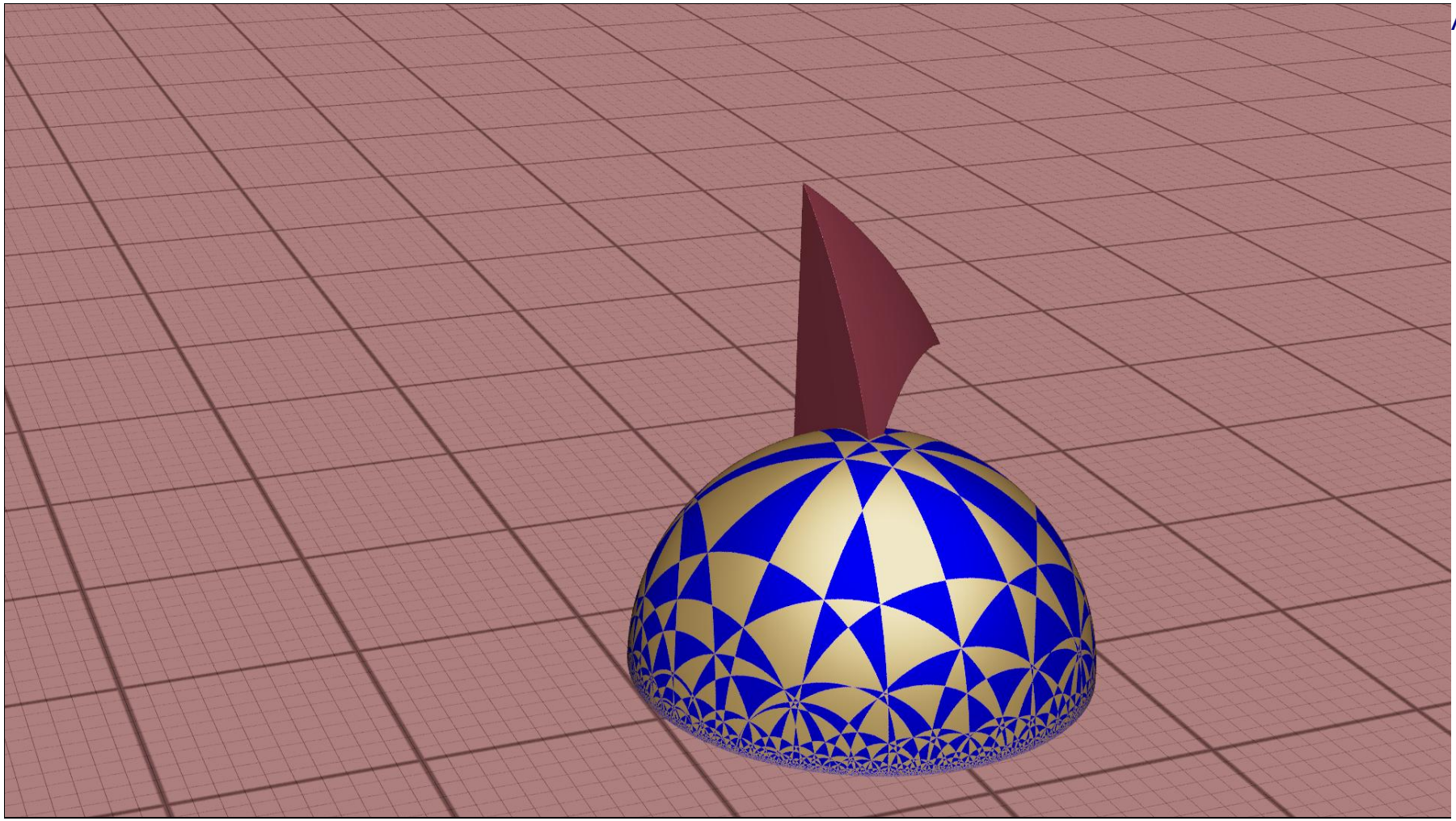
Half sphere cross section 8



Half sphere cross section 9

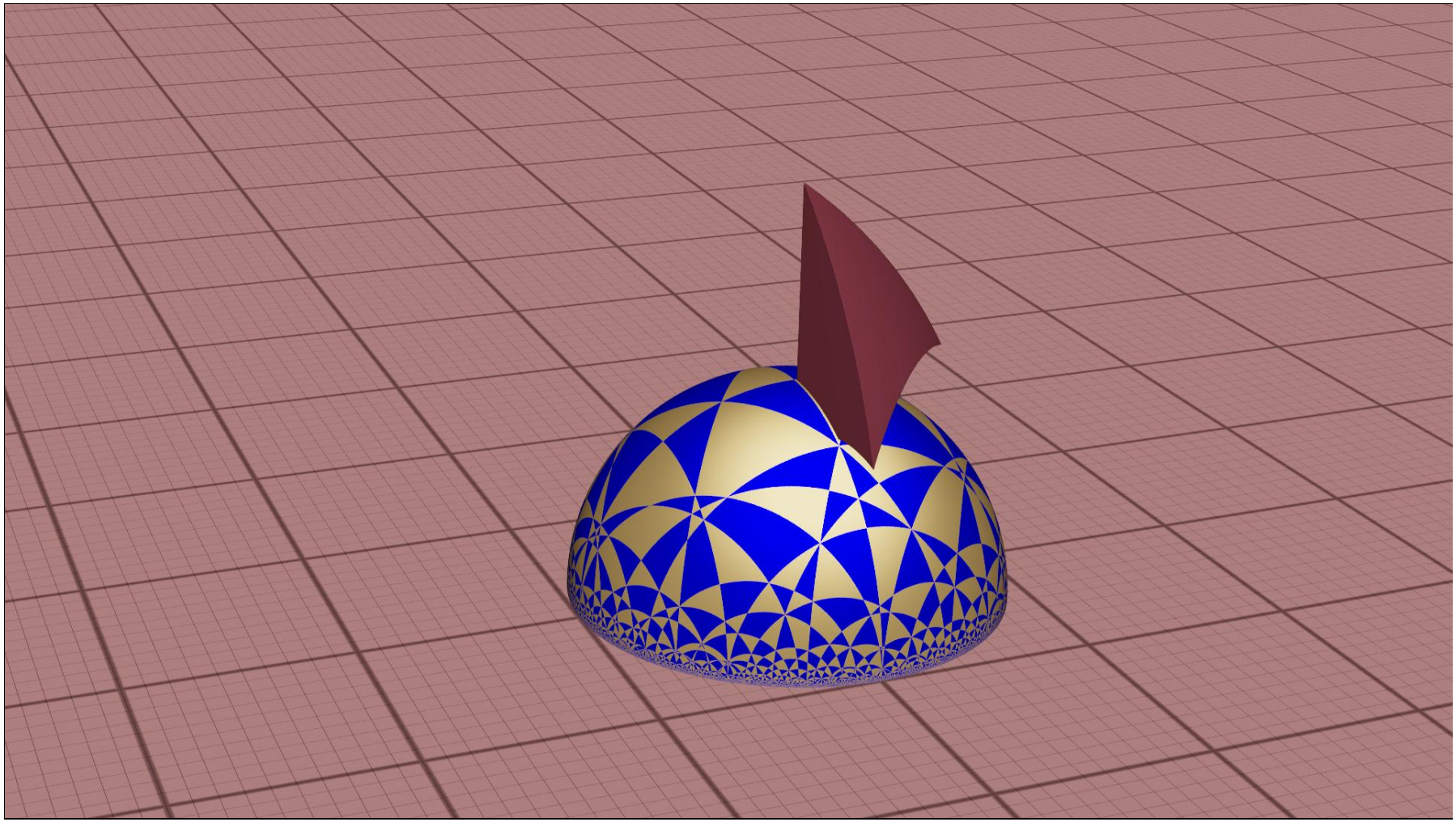


Half sphere cross section 10

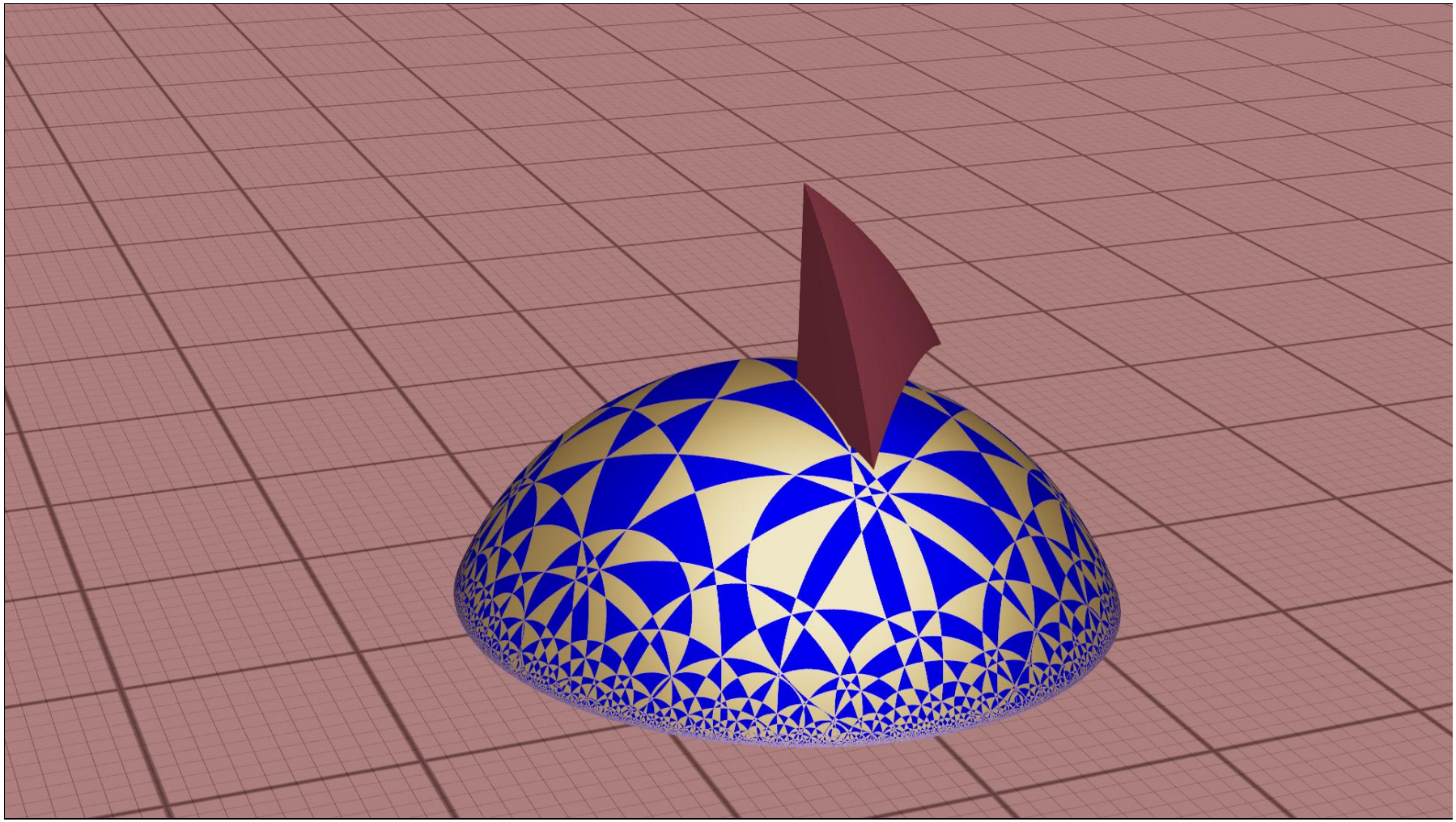


Animation

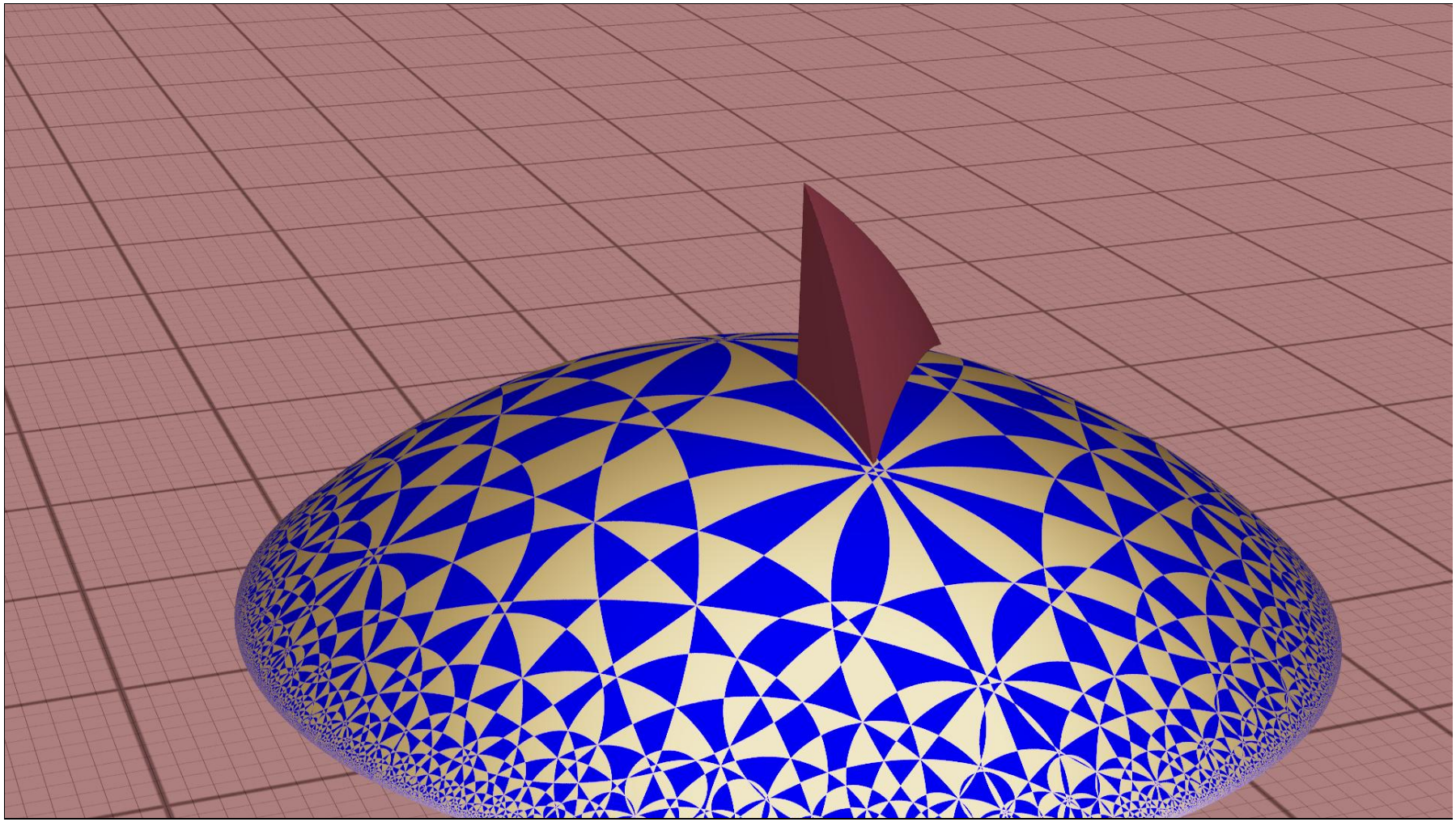
Plane to Lens to Horosphere



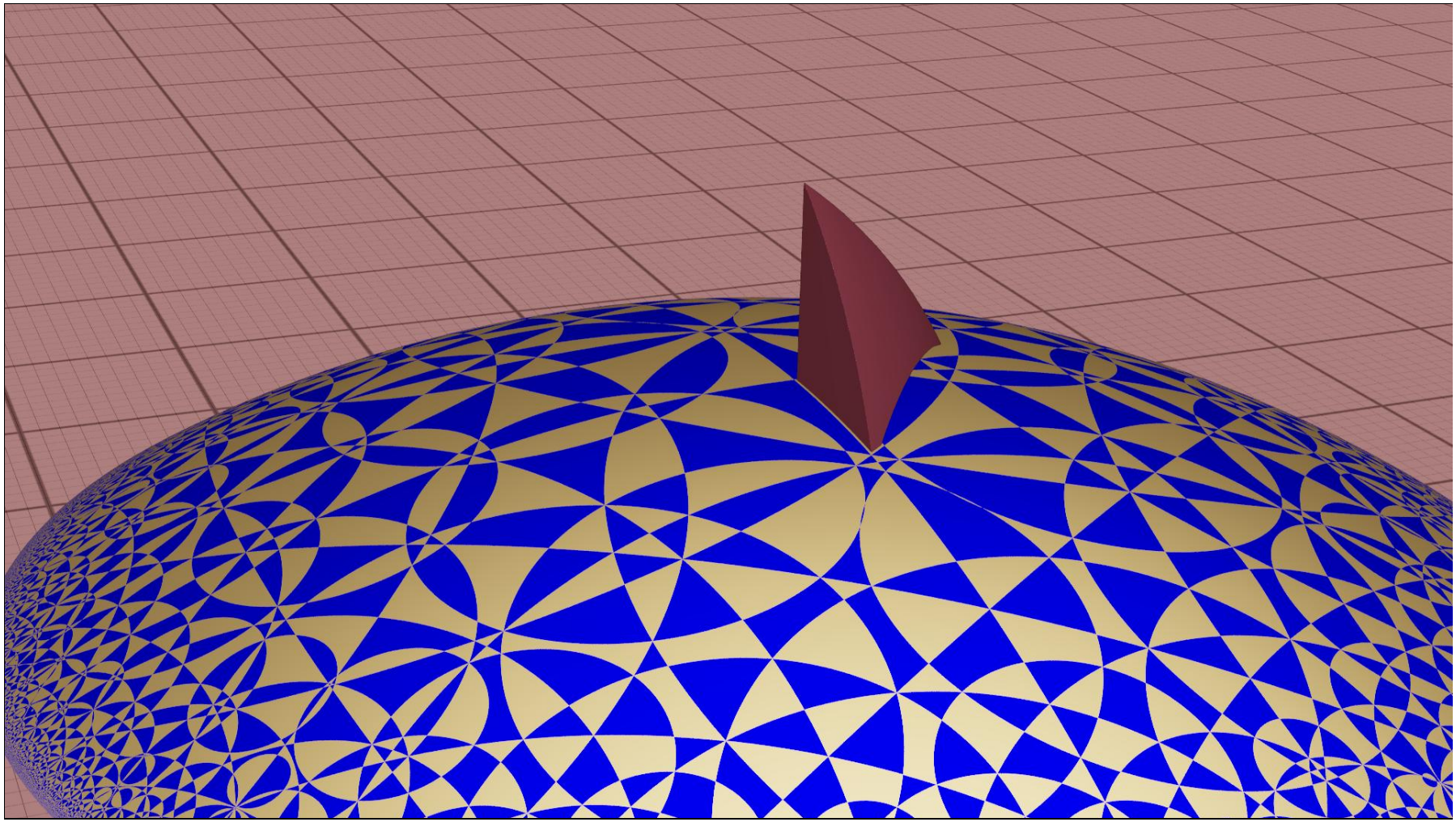
Plane to Lens to Horosphere 1



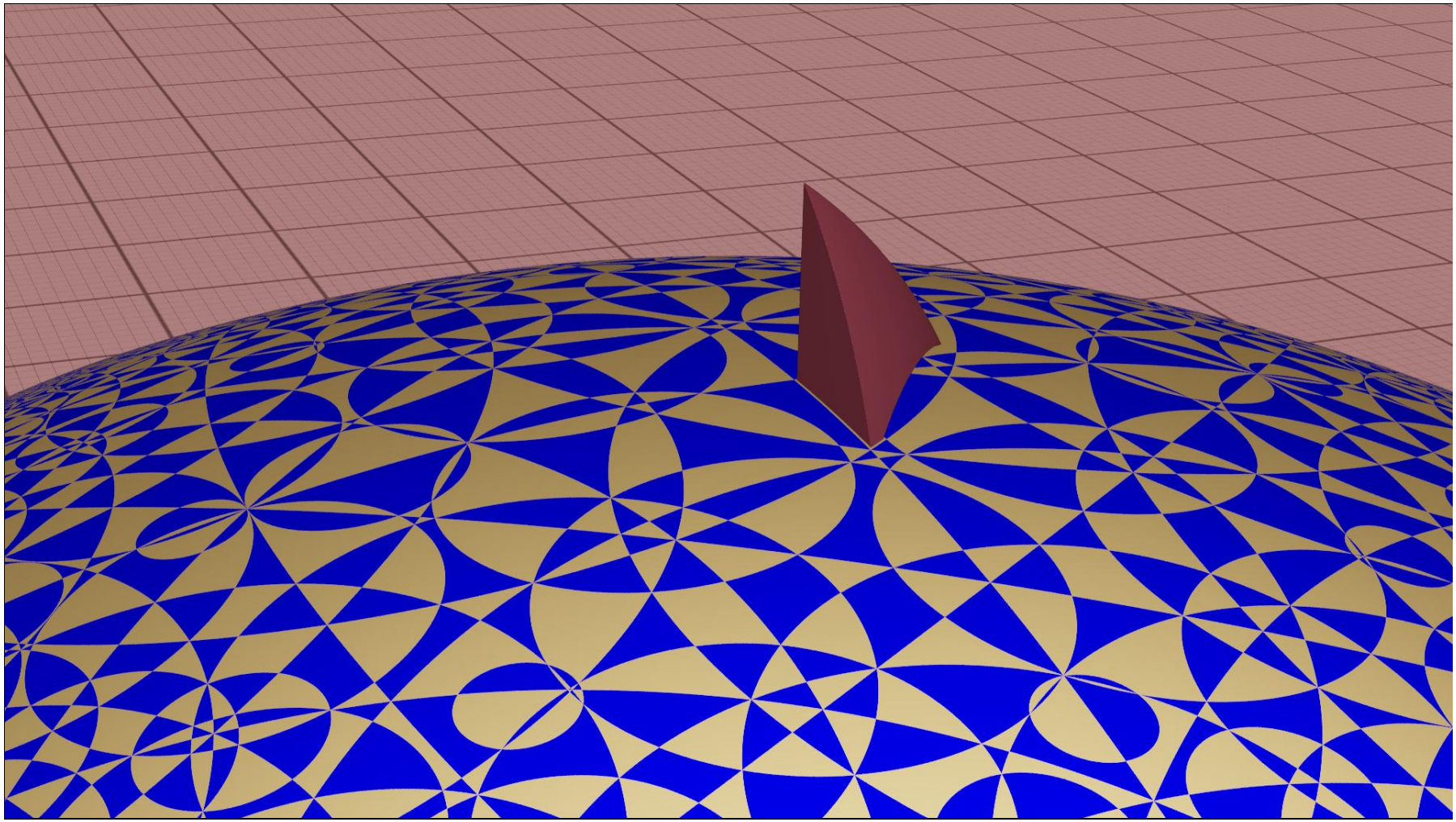
Plane to Lens to Horosphere 2



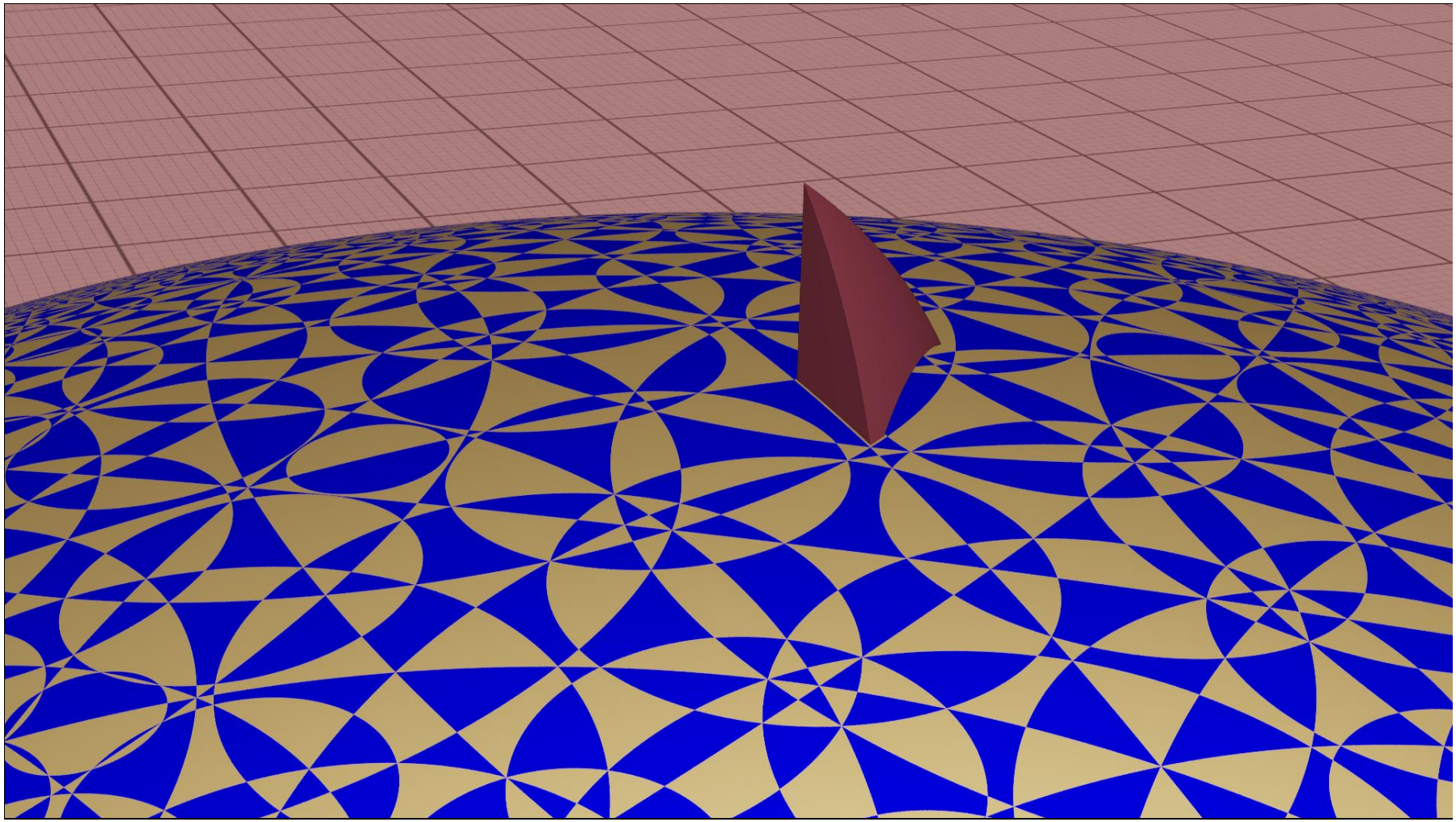
Plane to Lens to Horosphere 3



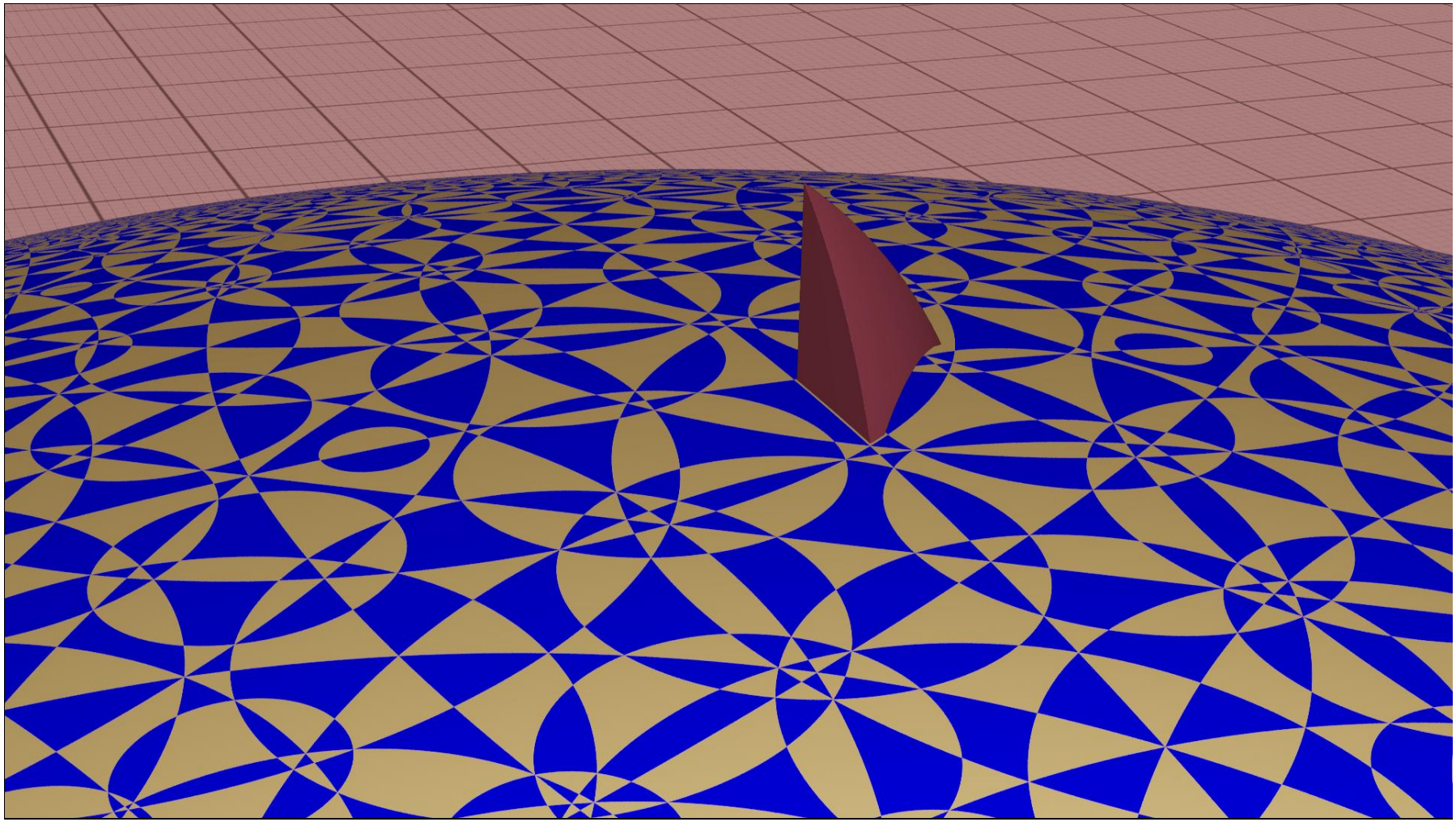
Plane to Lens to Horosphere 4



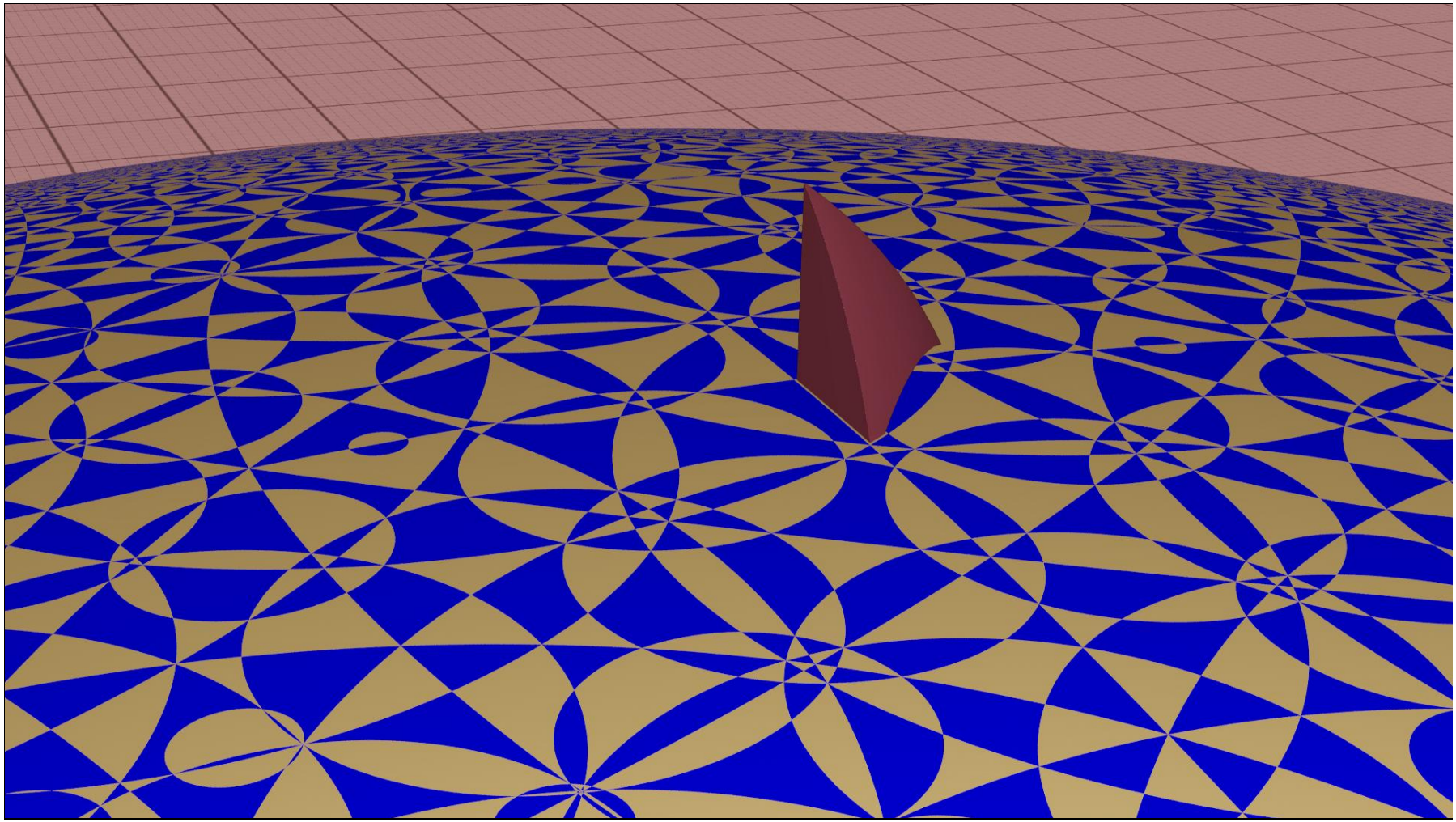
Plane to Lens to Horosphere 5



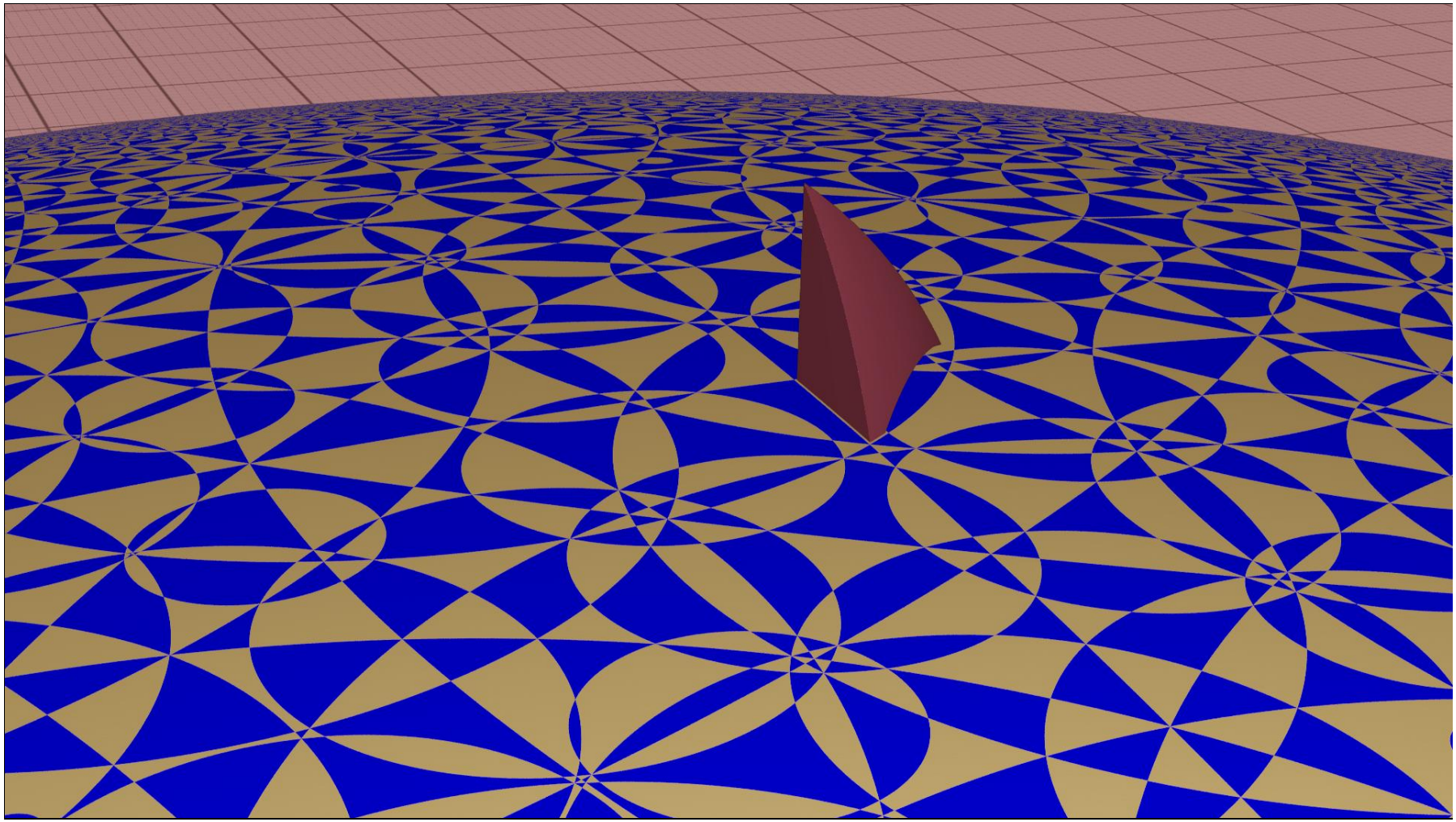
Plane to Lens to Horosphere 6



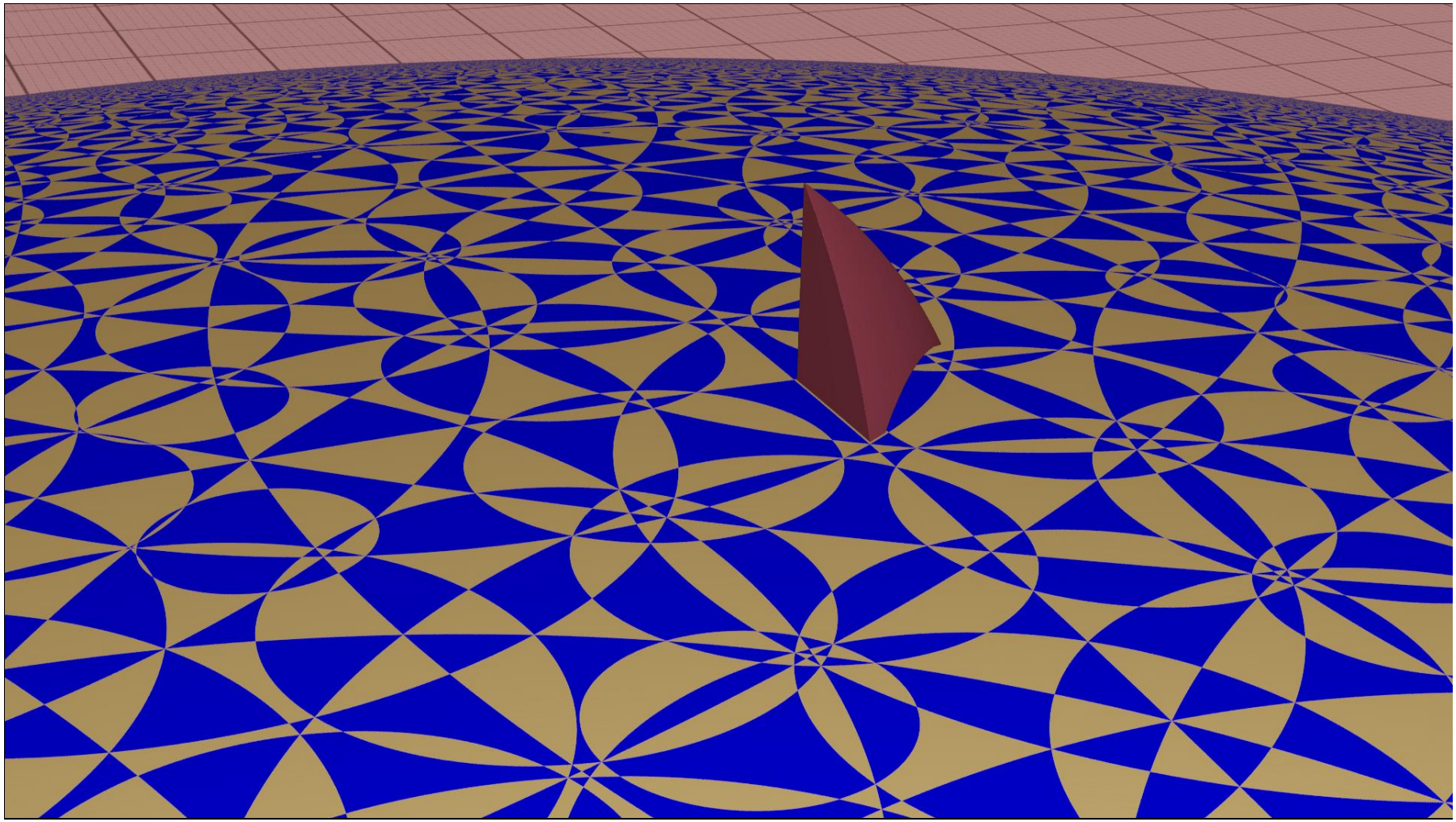
Plane to Lens to Horosphere 7



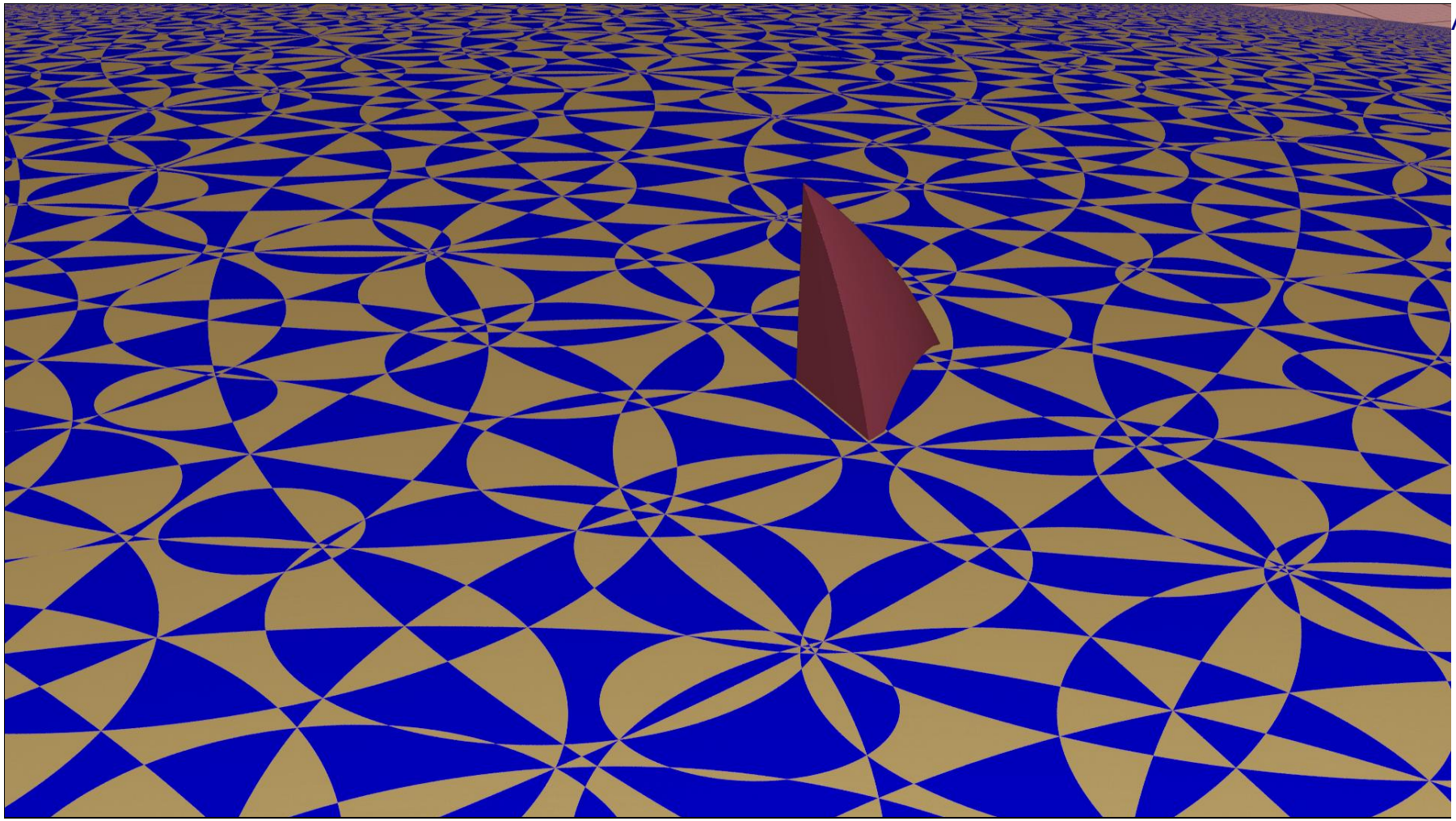
Plane to Lens to Horosphere 8



Plane to Lens to Horosphere 9

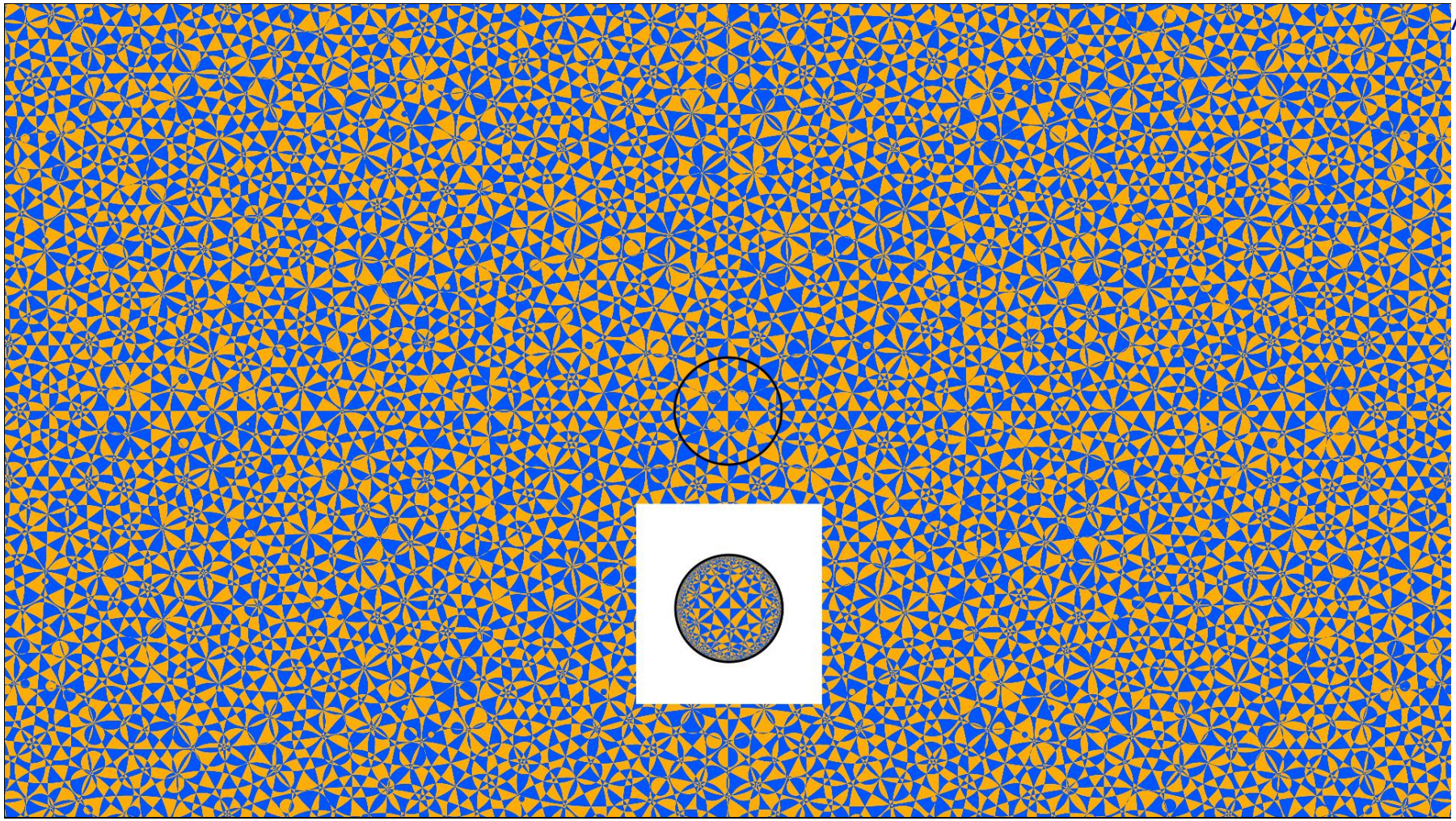


Plane to Lens to Horosphere 10



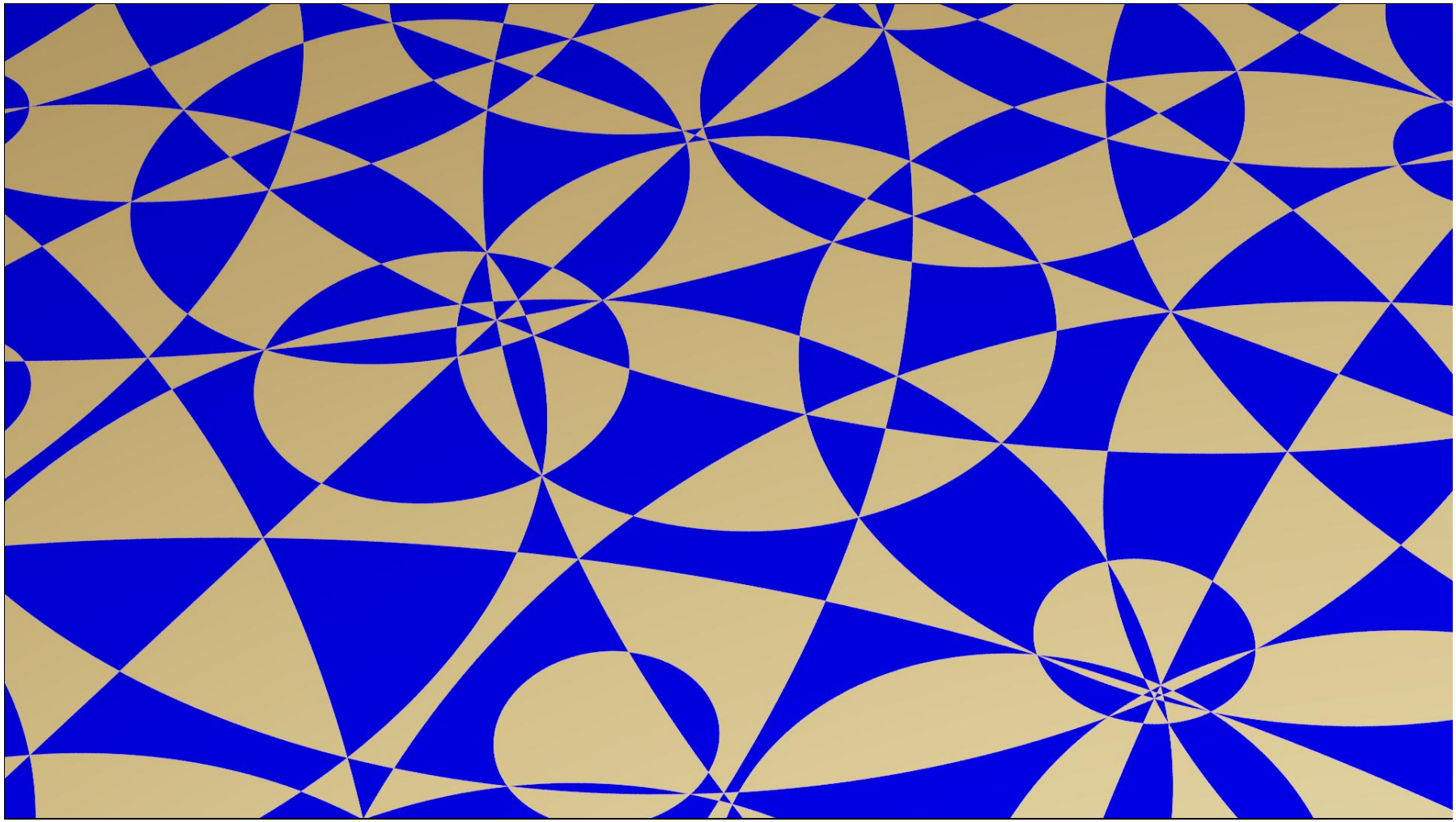
Animation

Plane to Lens to Horosphere 11

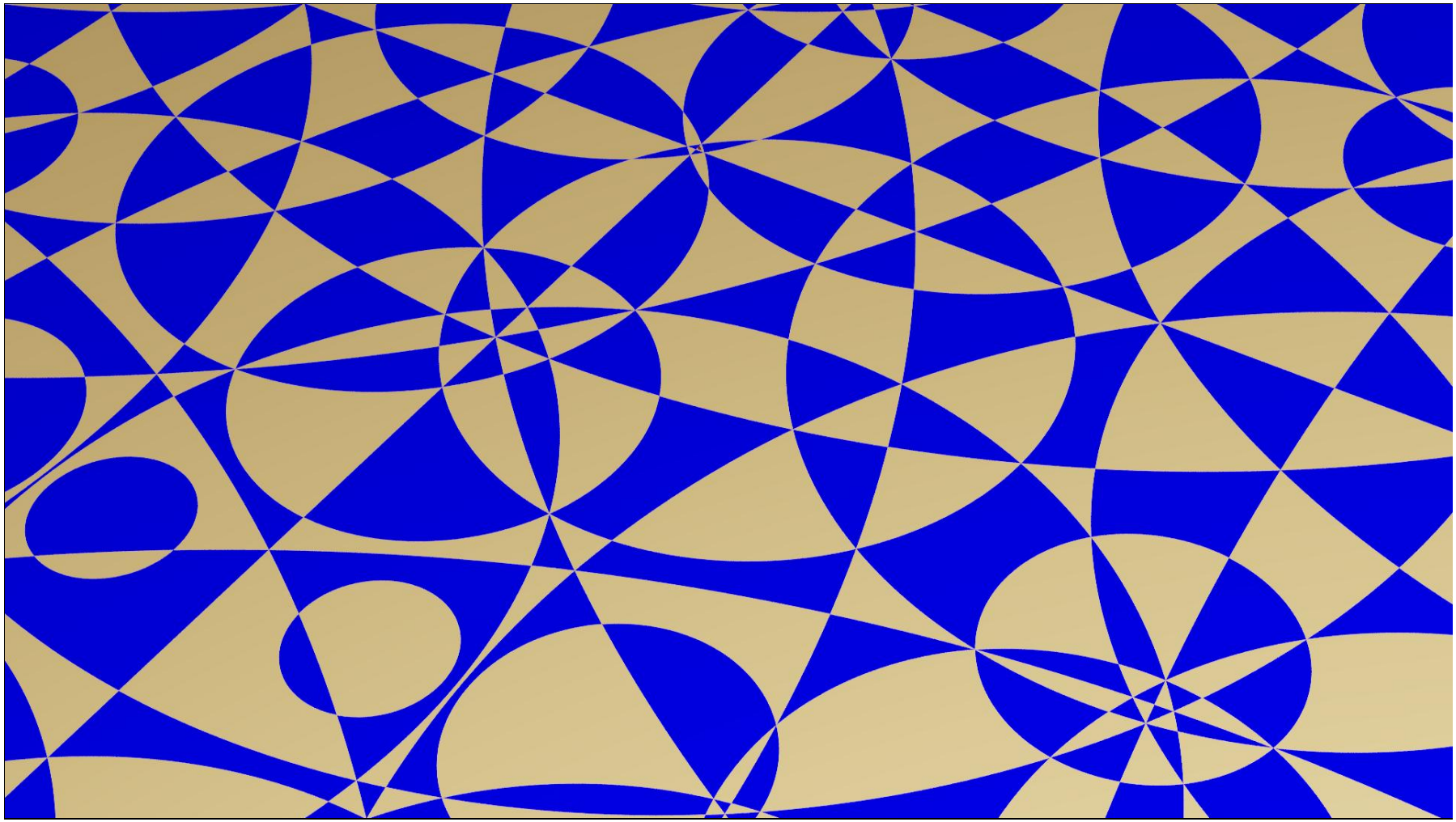


Animation

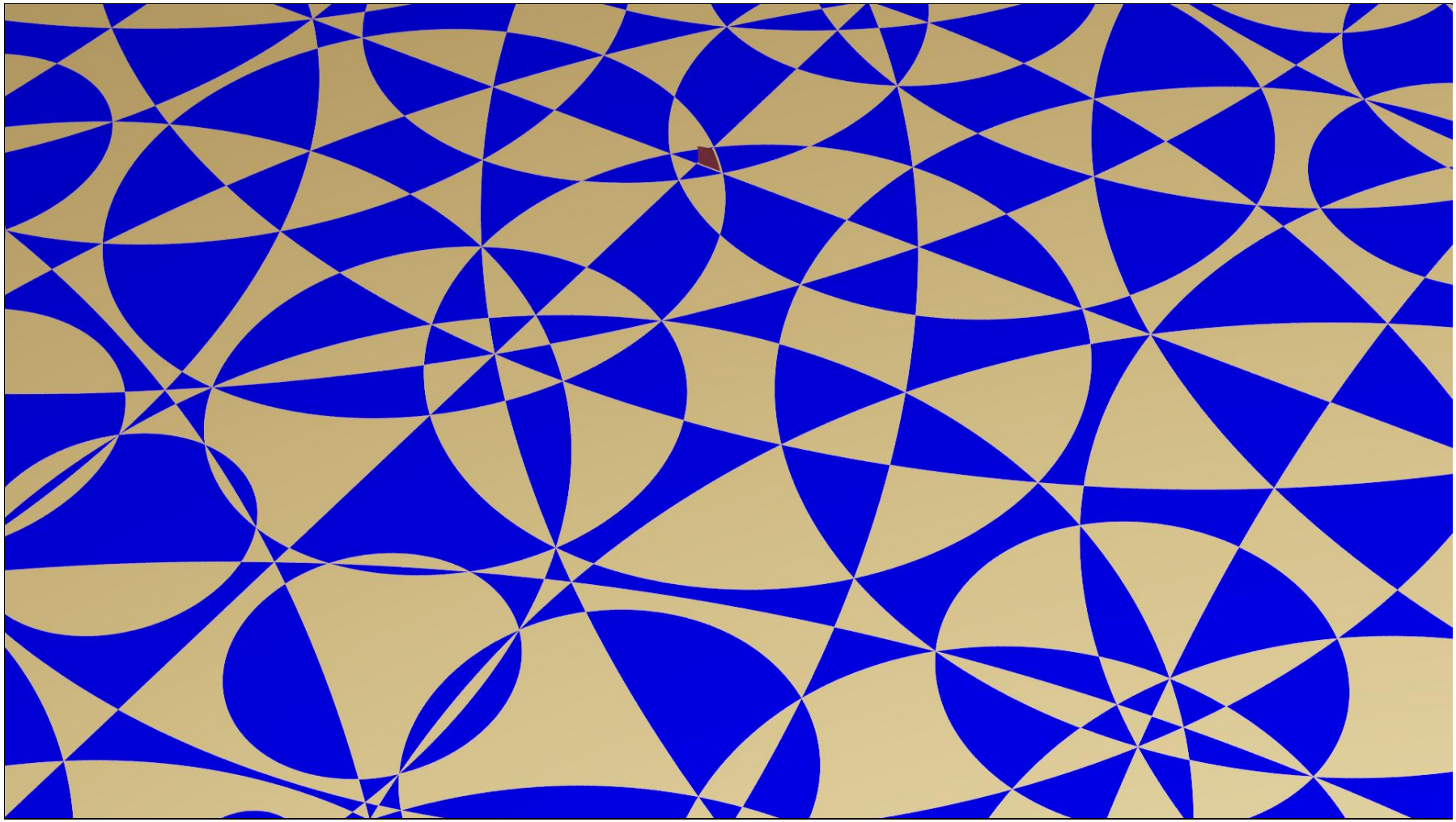
Horosphere Motion



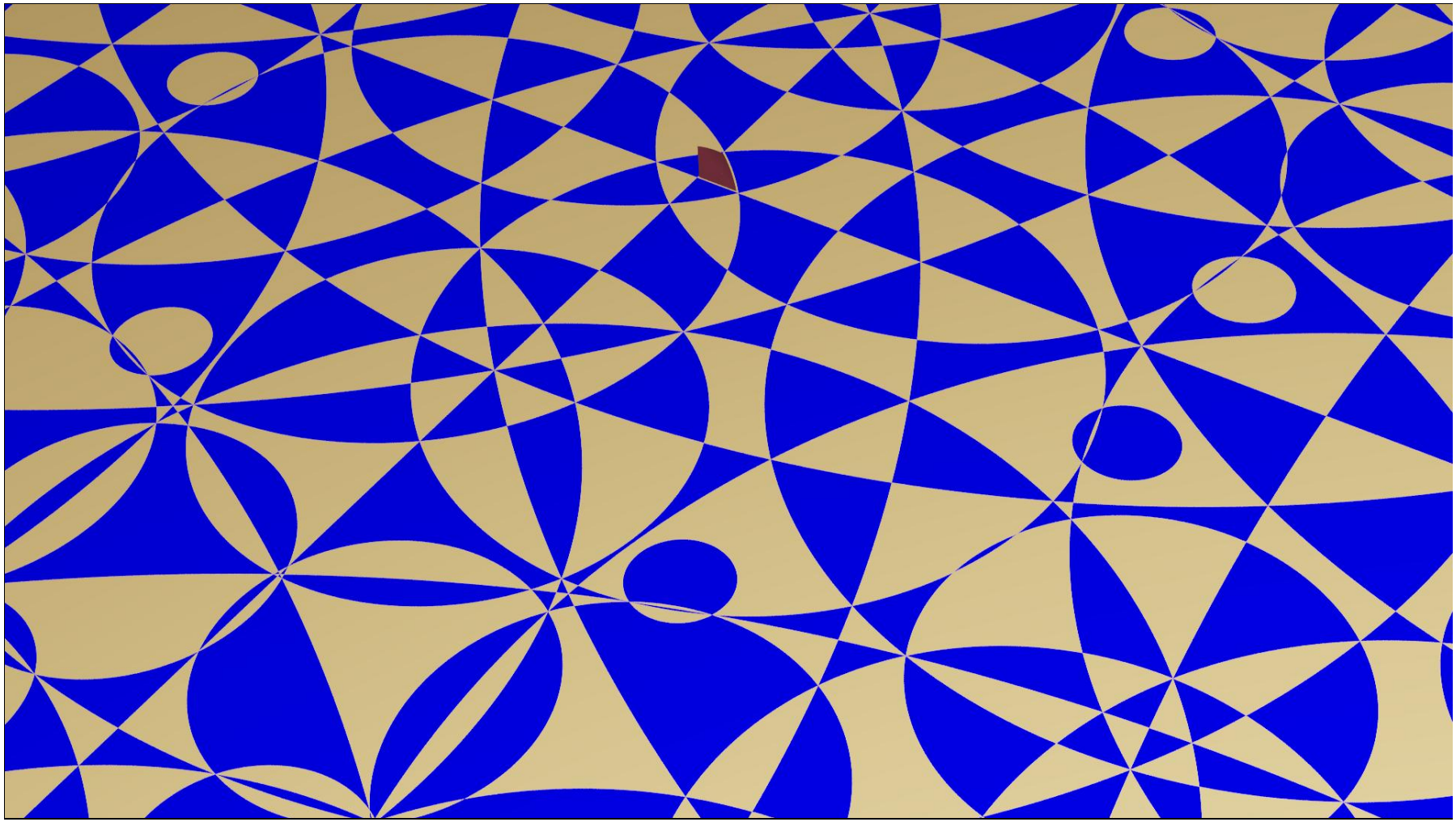
Horosphere Motion 1



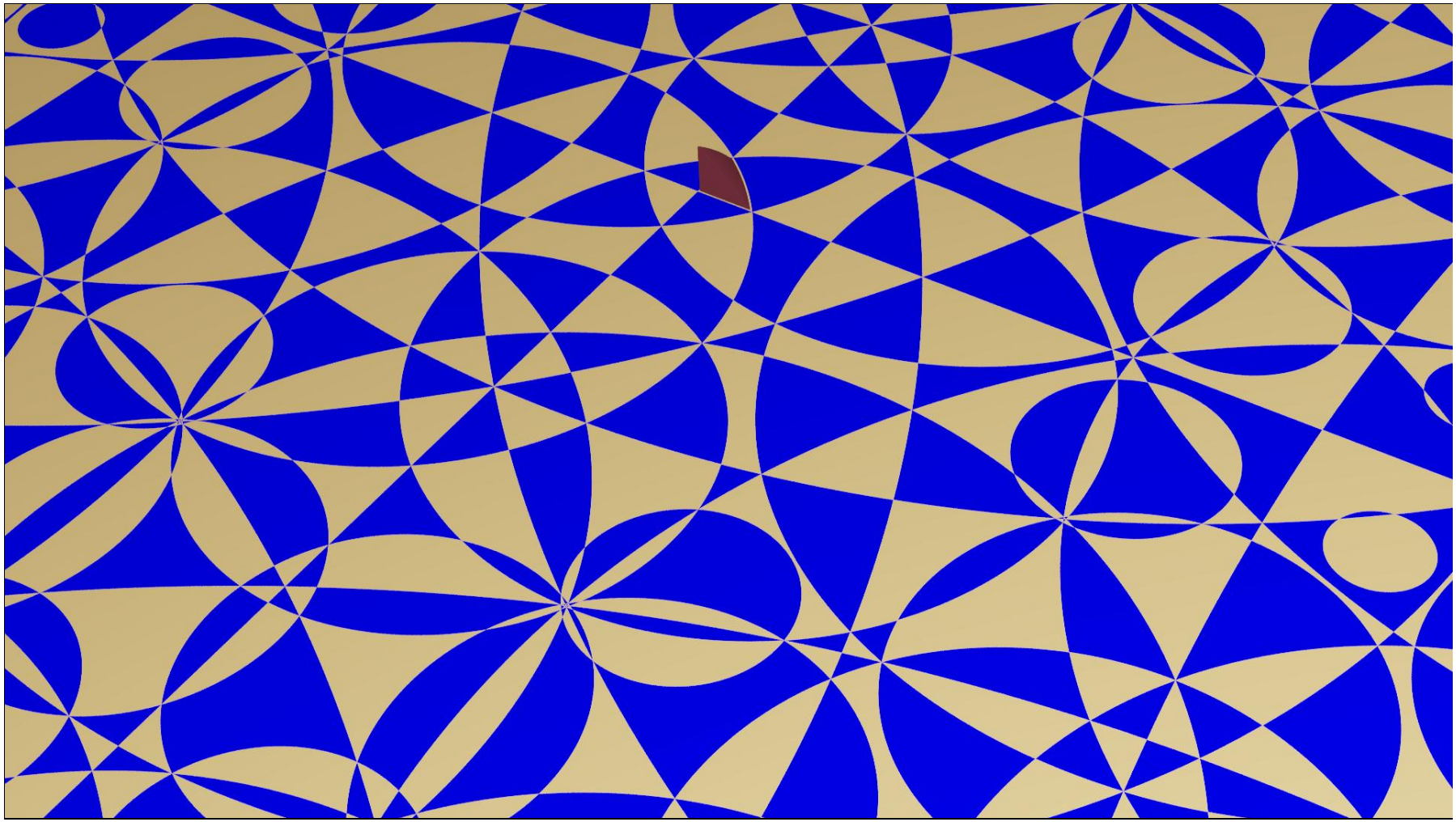
Horosphere Motion 2



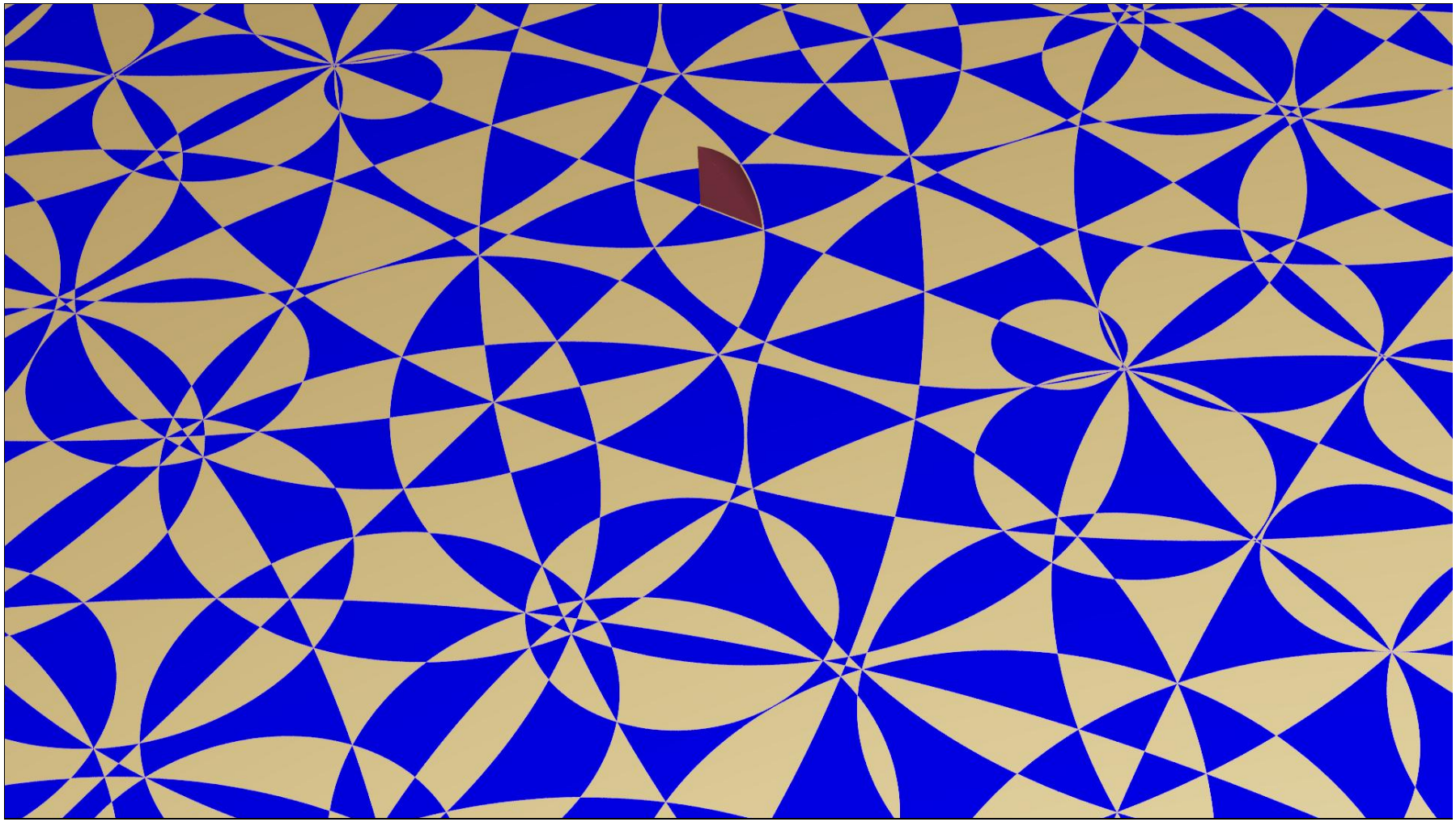
Horosphere Motion 3



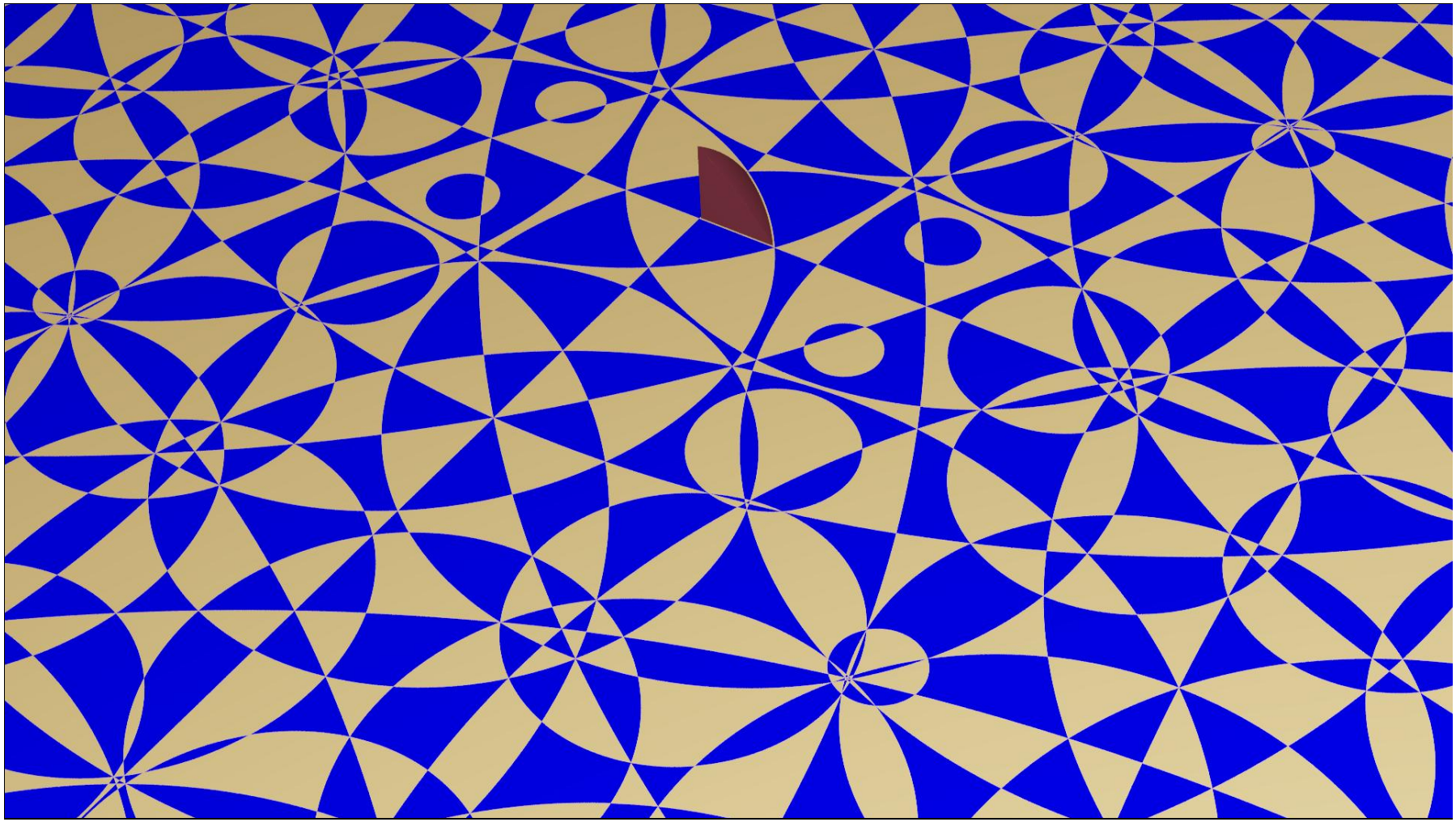
Horosphere Motion 4



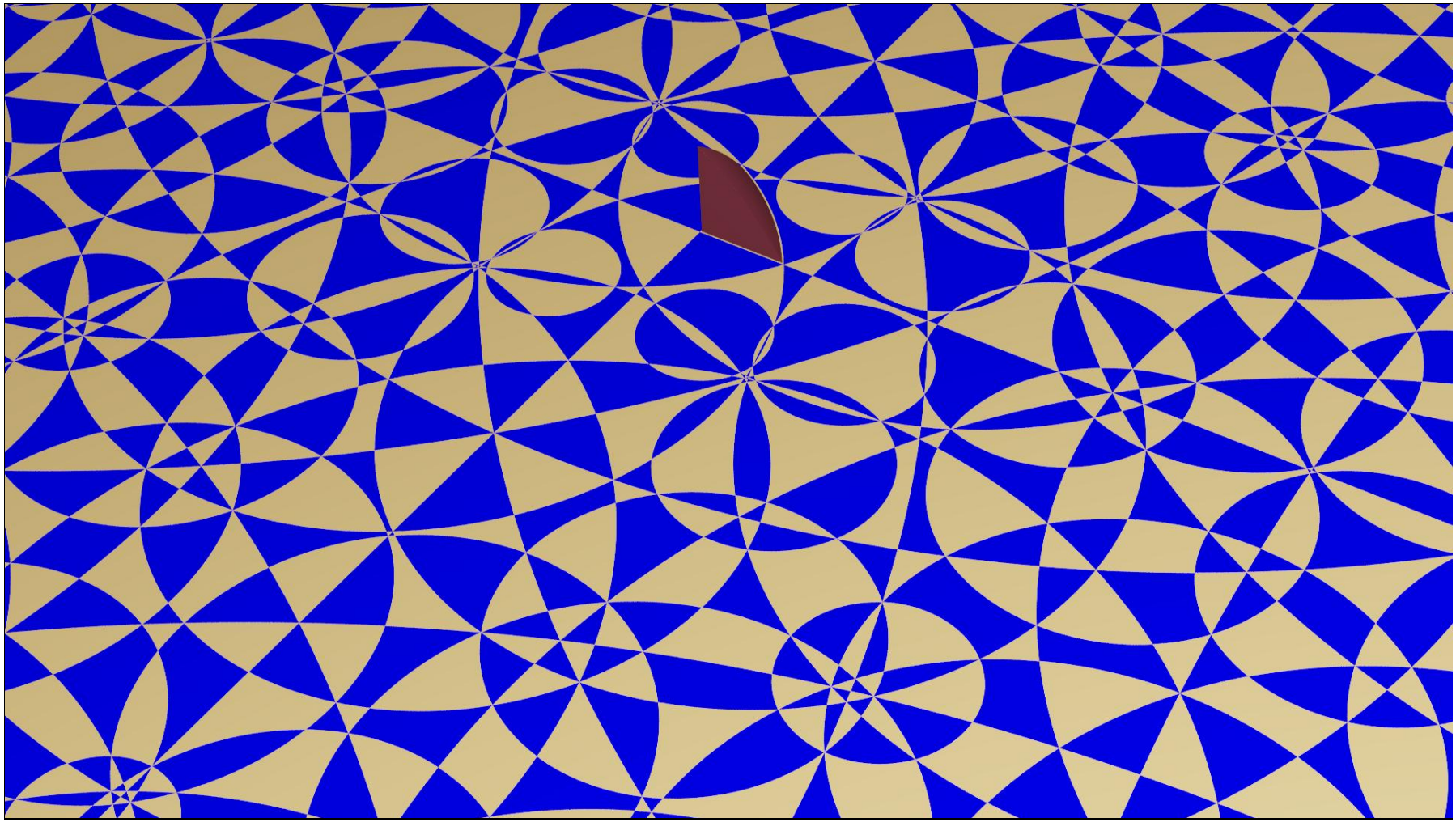
Horosphere Motion 5



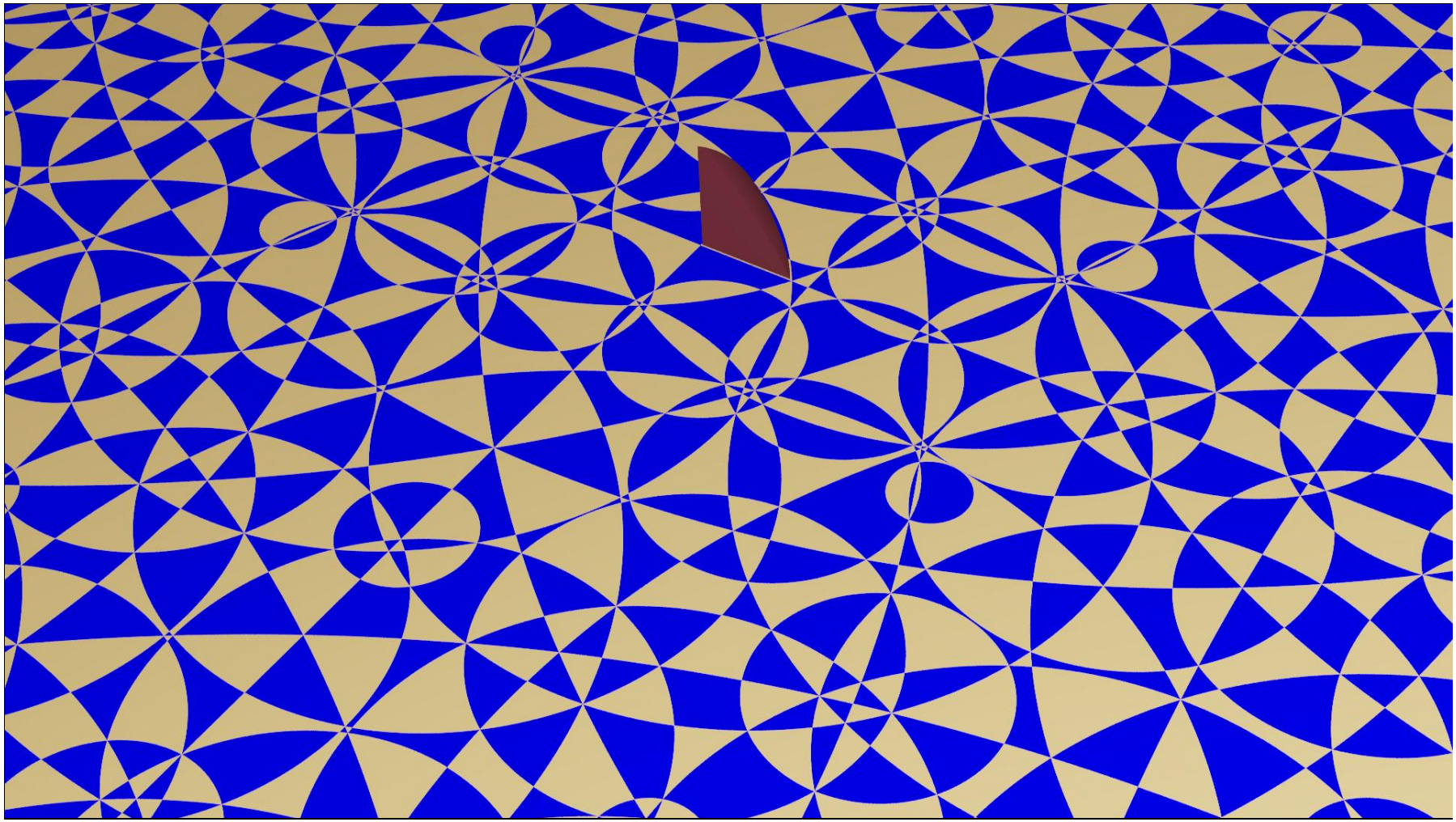
Horosphere Motion 6



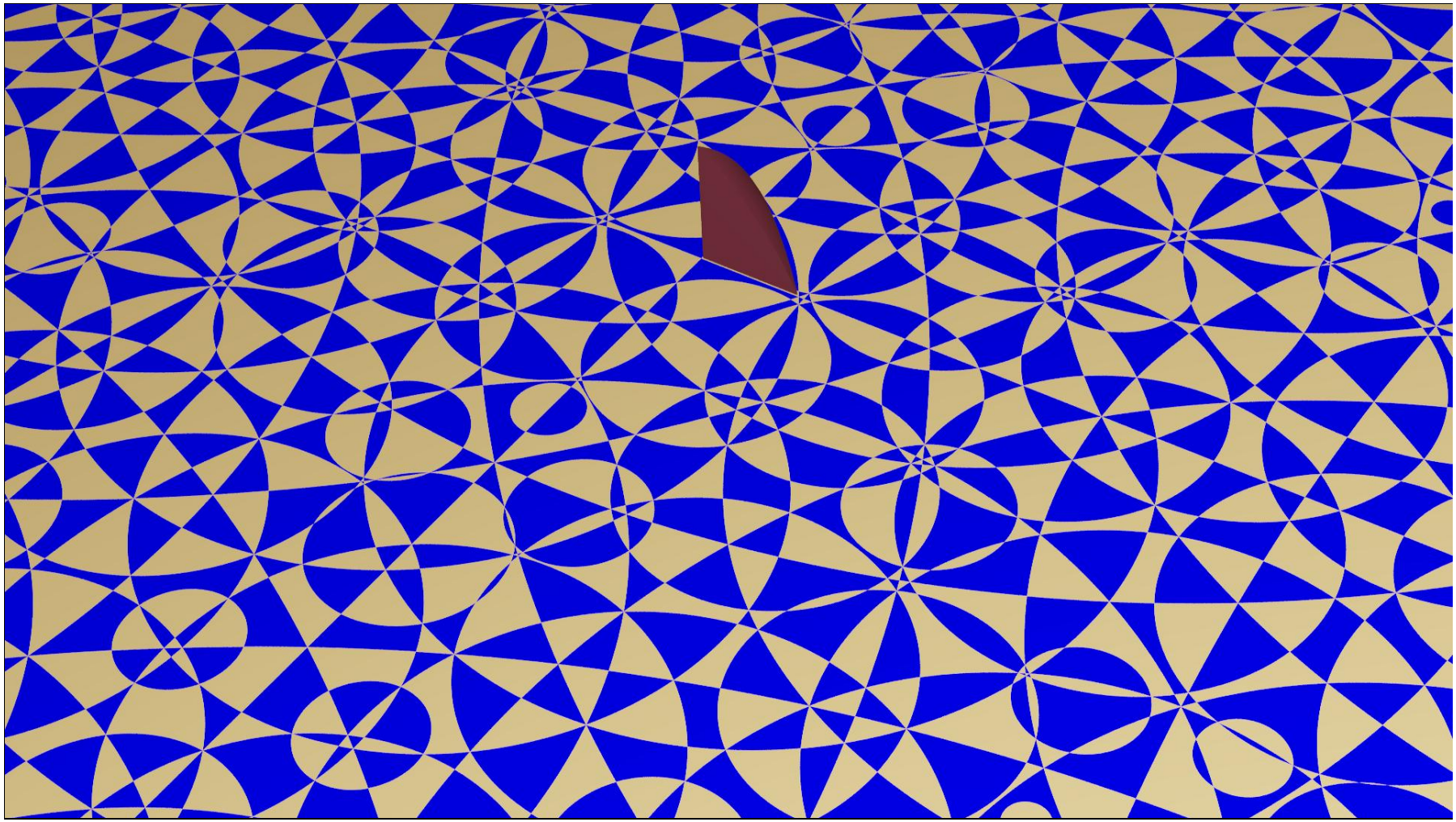
Horosphere Motion 7



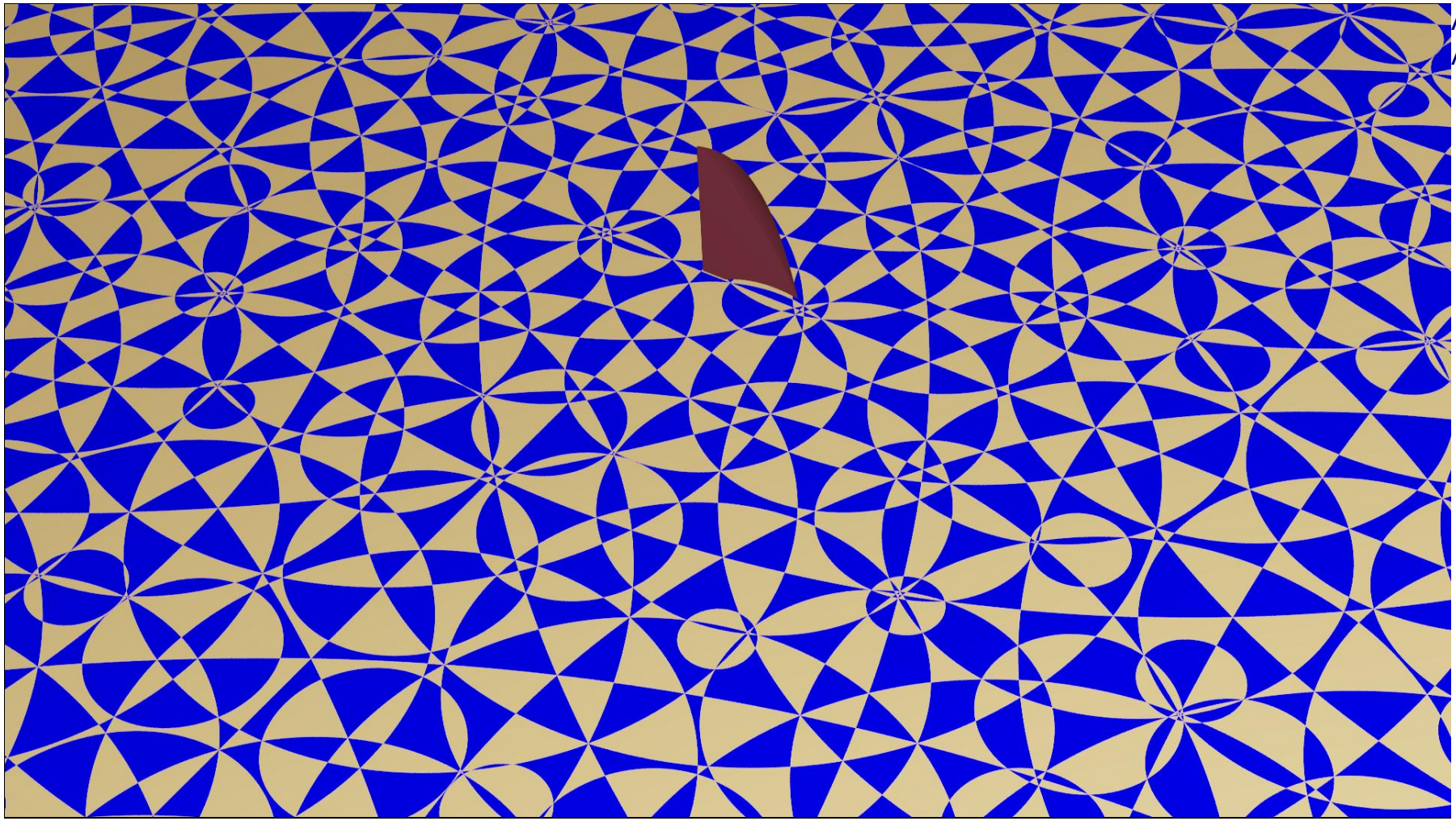
Horosphere Motion 8



Horosphere Motion 9

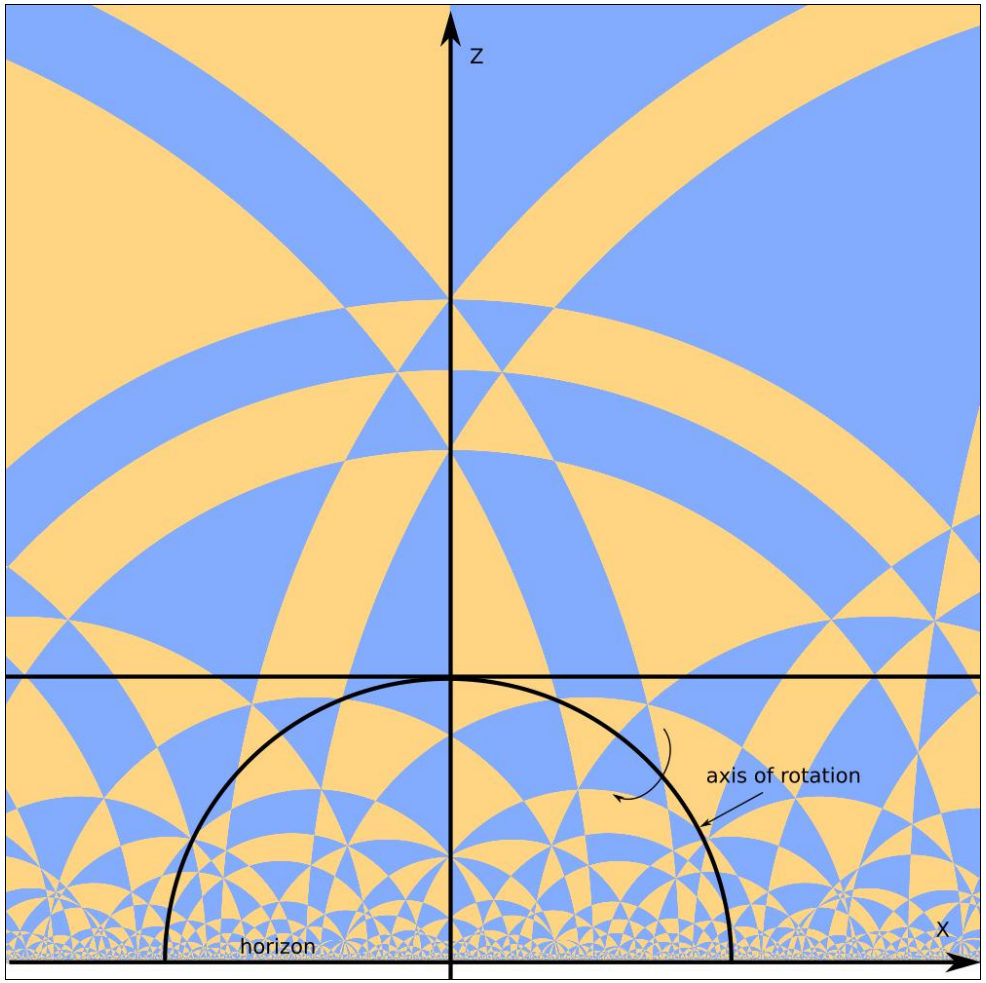


Horosphere Motion 10



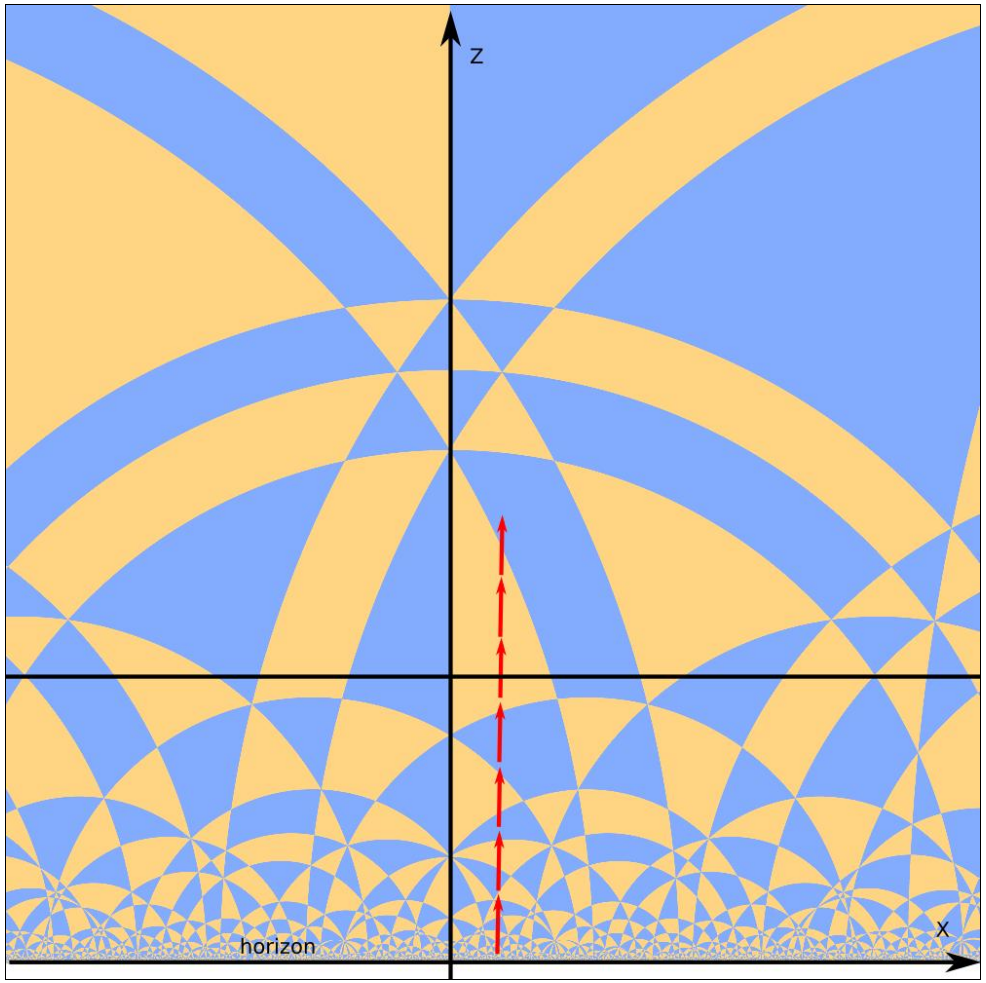
Animation 1.
Animaton 2.

Tiling Rotation



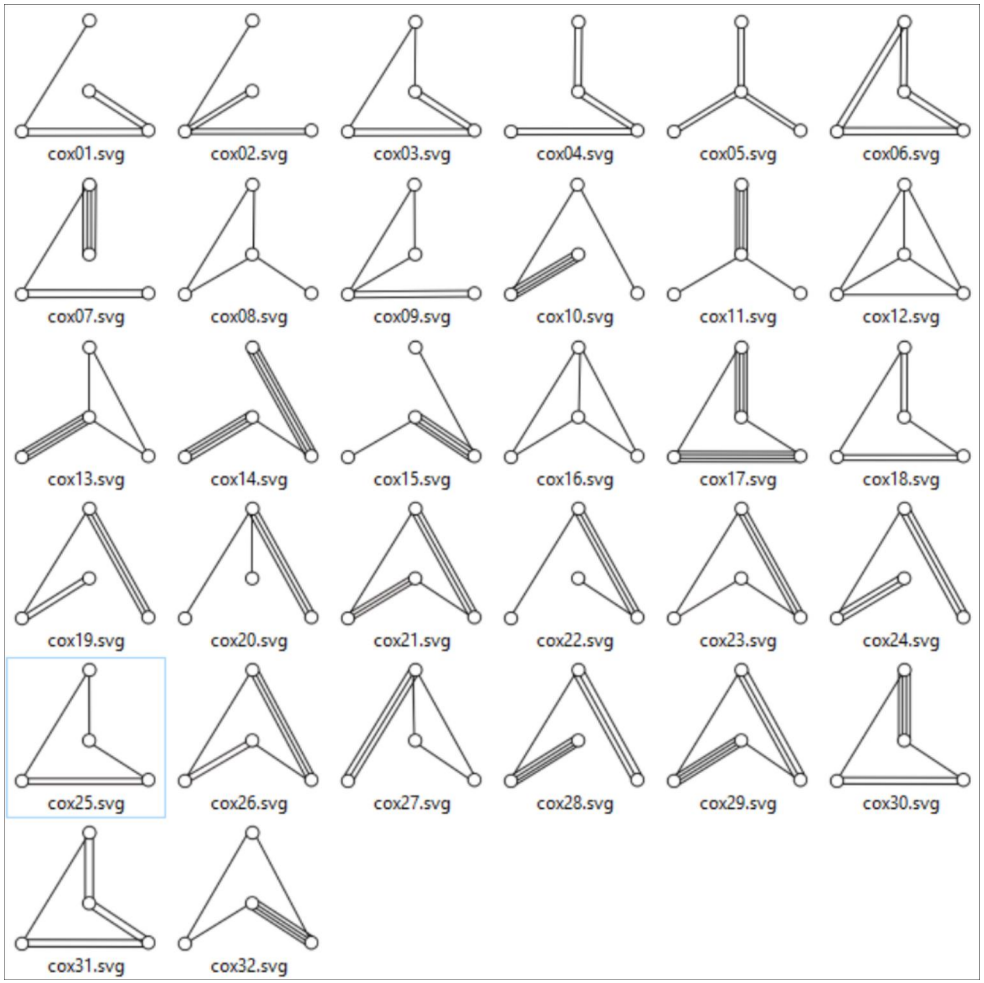
Tiling is rotated about a hyperbolic straight line.
animaton 1.
animaton 2.

Tiling Translation



Animation.

Different Hyperbolic Tilings



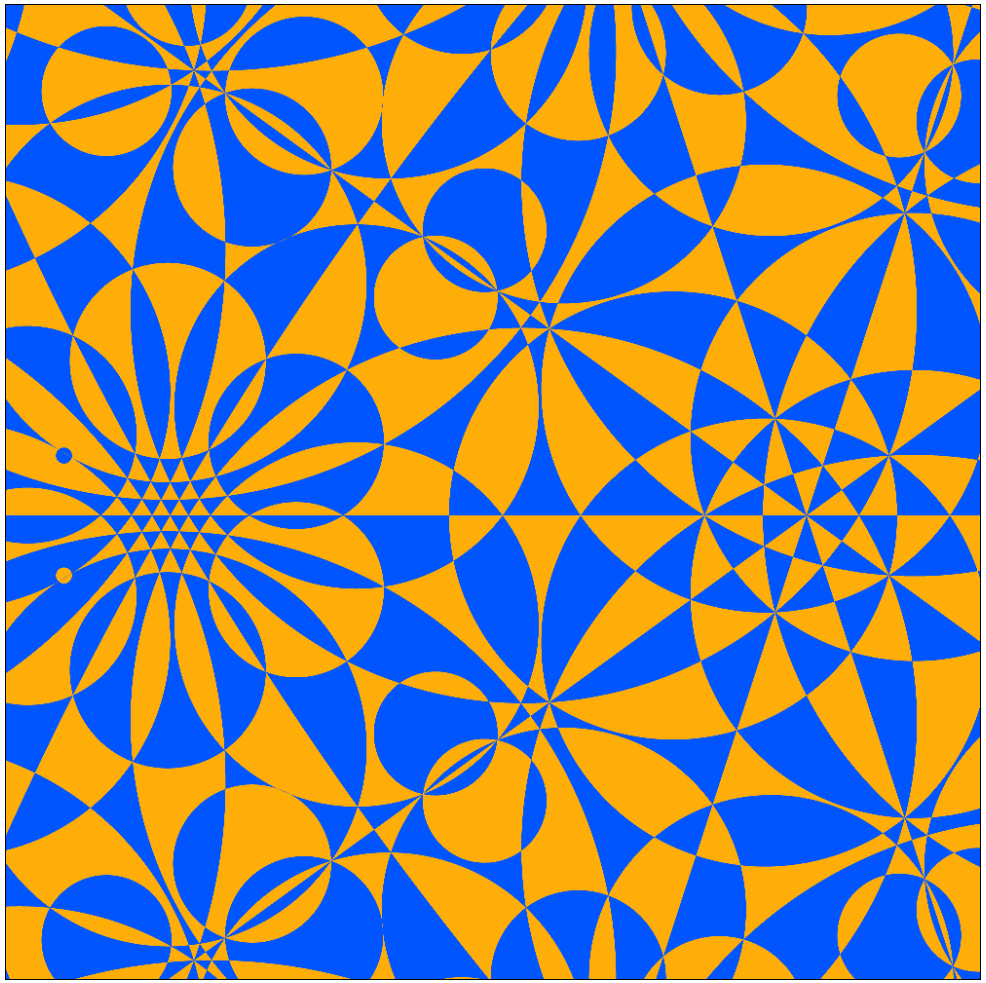
There are 32 compact hyperbolic tetrahedra with kaleidoscopic angles.

There is infinite number of non compact tetrahedra (tetrahedra with vertices beyond infinity).

There is infinite number of more complex polyhedra.

A lot of possibilities for fun.

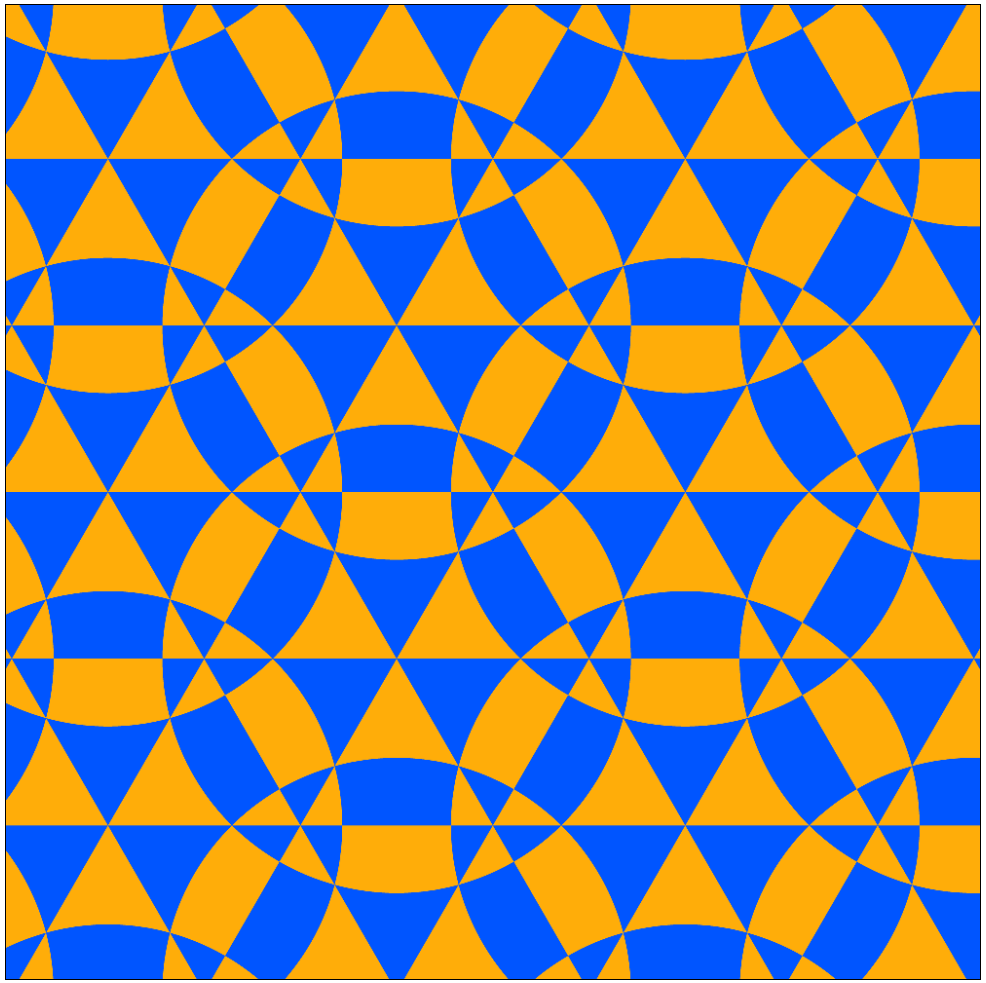
Tetrahedron example 1



This tetrahedron has icosahedral vertex $\star 532$ and ideal vertex $\star 333$.

Animation.

Tetrahedron example 2



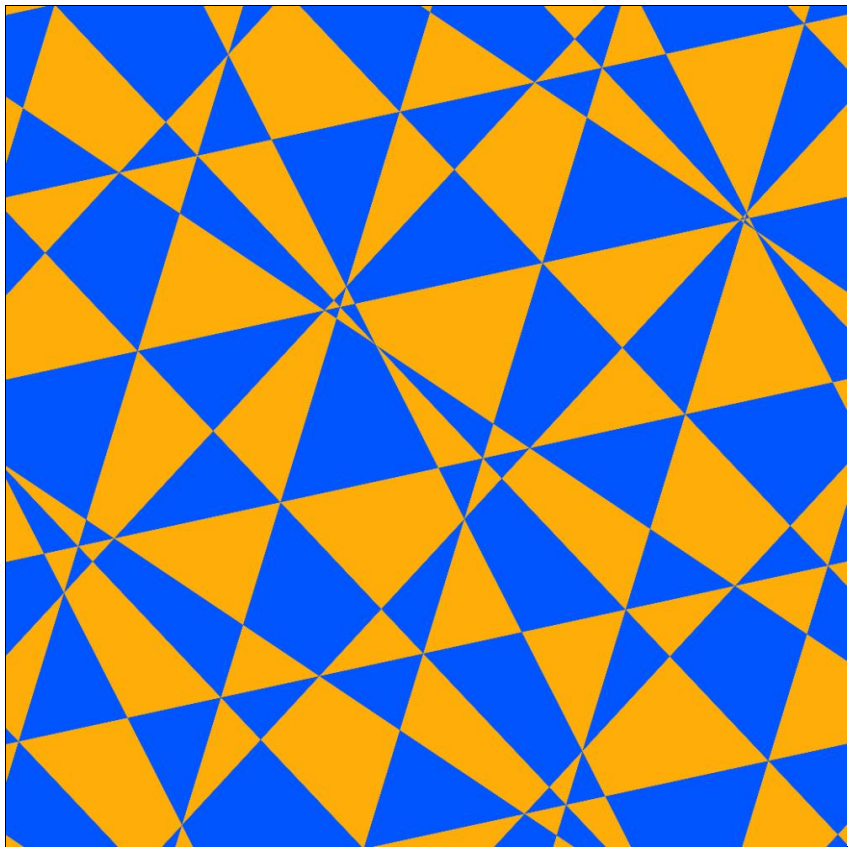
Tetrahedron has ideal vertex $\star 333$.

Horosphere is moving away from that vertex.

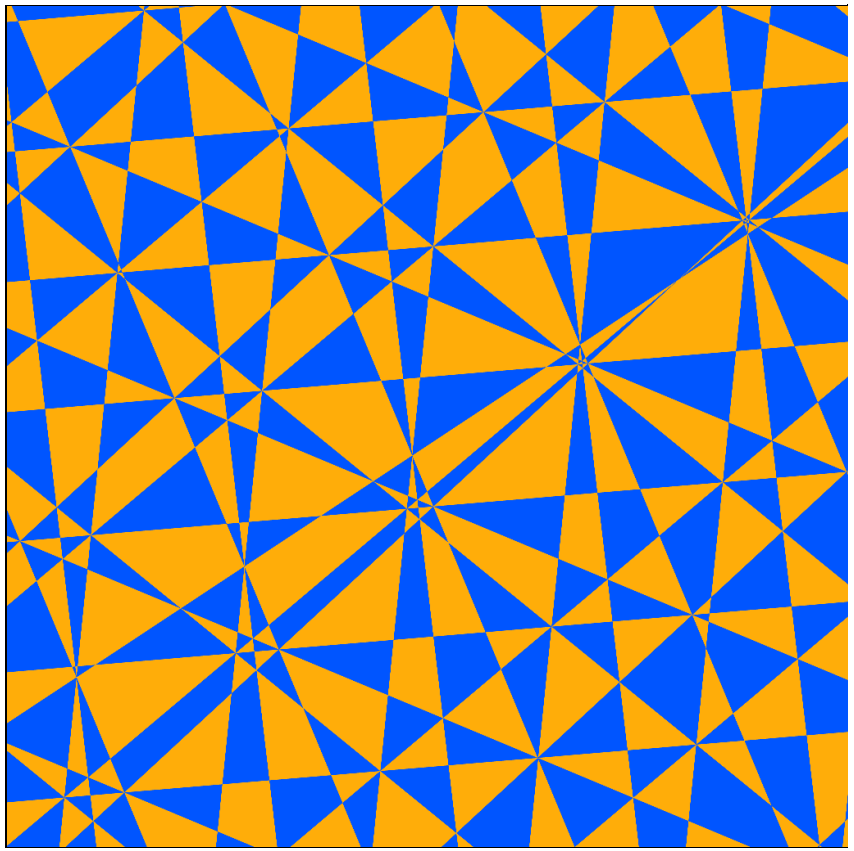
Animation.

Other Cross Sections

Slicing euclidean tiling wih plane also produces interesting results

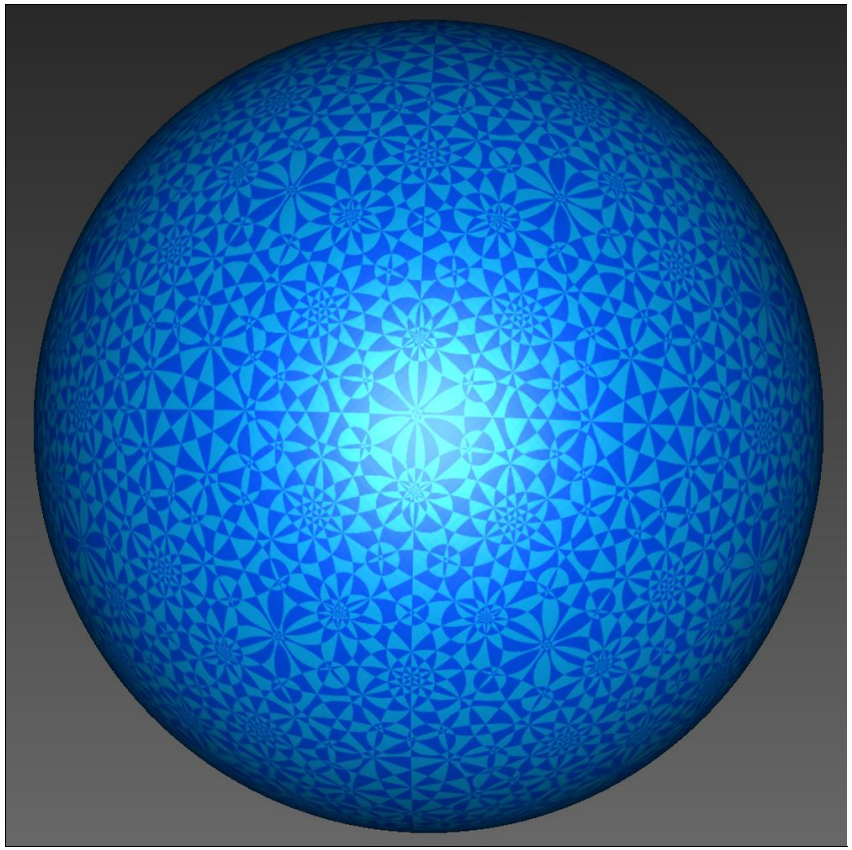


Animation

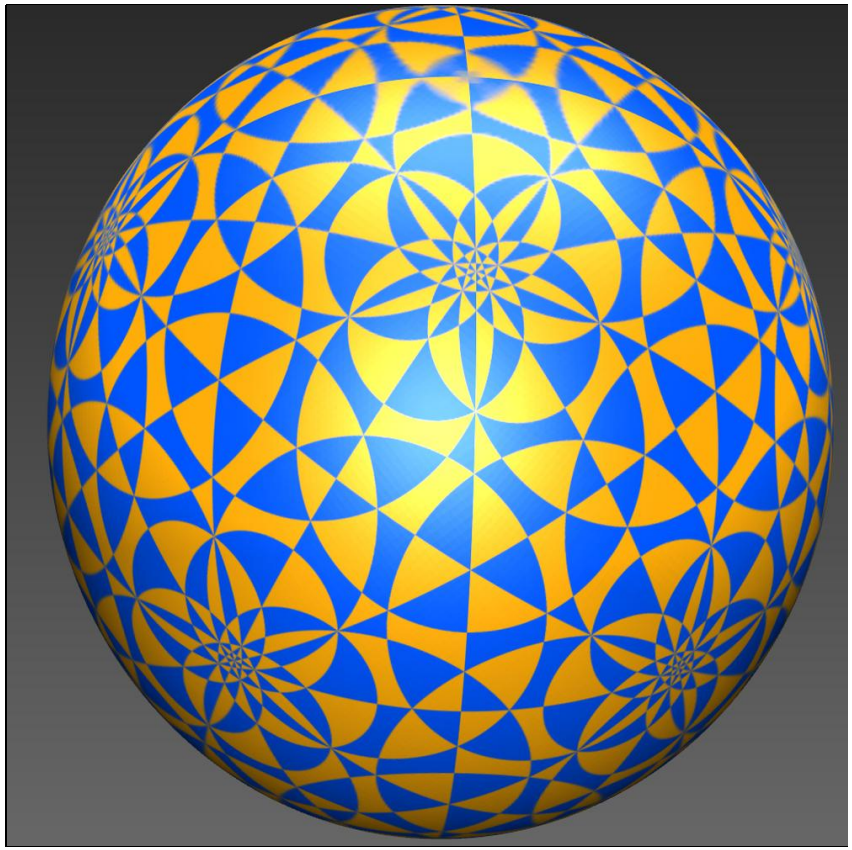


Animation

Spherical Cross Sections

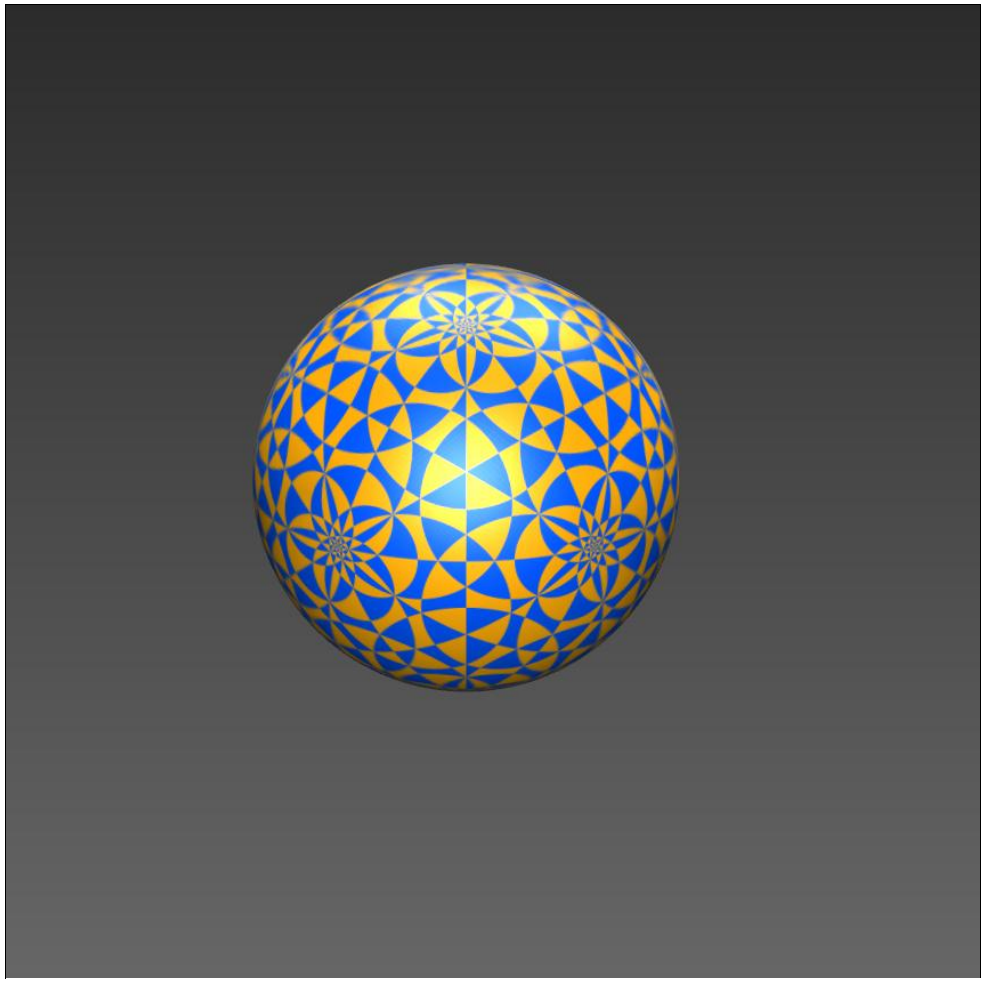


VRML.



VRML.

3D Printing



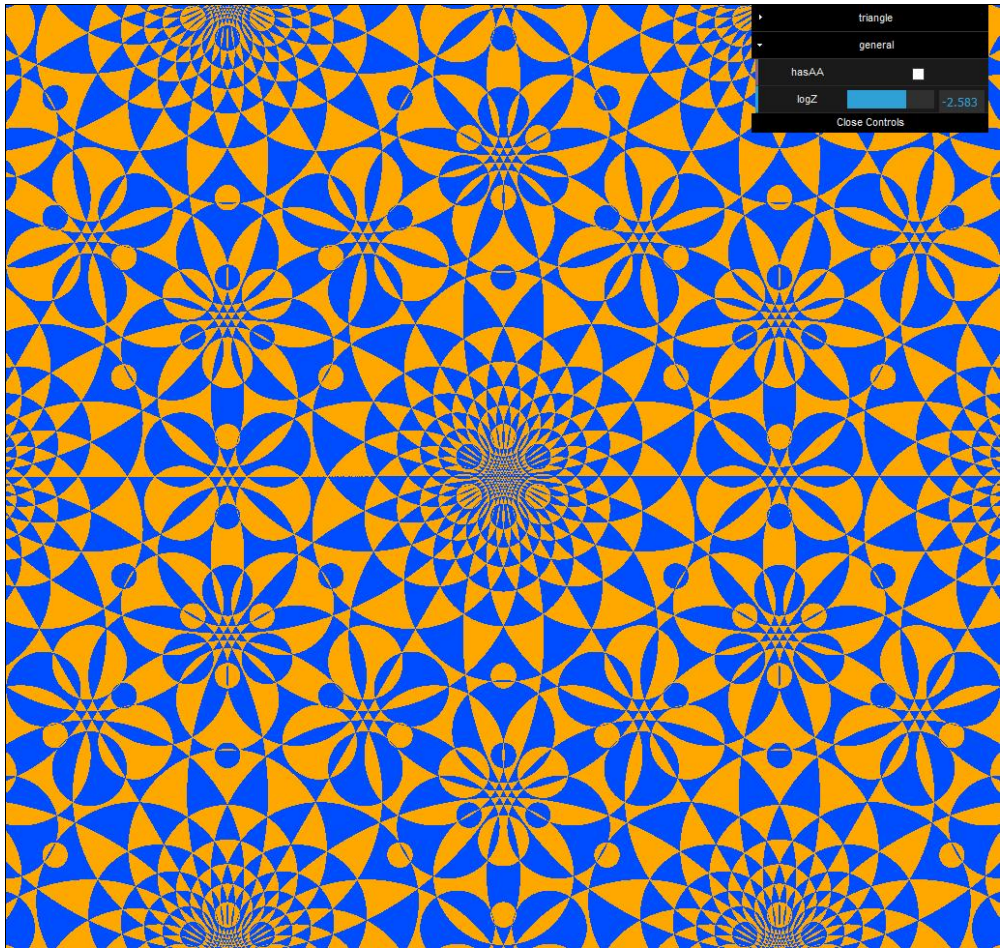
3D printing involves creation of spherical color map and applying it to a shape (sphere).

The modeling was done in **ShapeJS** - online tool for making 3d printable models via script.

The 3D printing was done by Shapeways.



Real Time Rendering with WebGL



WebGL is web browser API which allow web pages run graphics program directly on users GPU (graphics card).

WebGL example

The End



Online adress of the talk
http://bulatov.org/math/18.04.11/